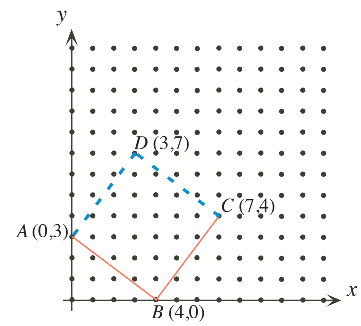
Problem of the Week
Grade 7 and 8

A Griddy Performance
Solution



Problem

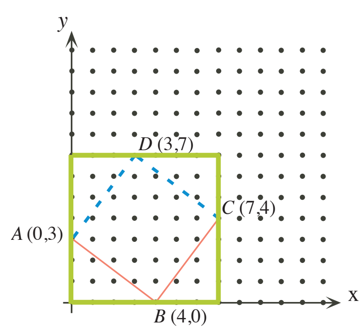
Three of the vertices of square $ABCD$ are located at A(0,3), B(4,0), and C(7,4).

(a) Determine the coordinates of the fourth vertex, D.

(b) Determine the area of square $ABCD$.

Solution

To determine the coordinates of D, observe that to get from A to B, you would go down 3 units and right 4 units. To get from B to C, you move 3 units to the right and then 4 units up. Continuing the pattern, go up 3 units and left 4 units you get to D(3,7). Continuing, as a check, go left 3 units and down 4 units, and you arrive back at A.

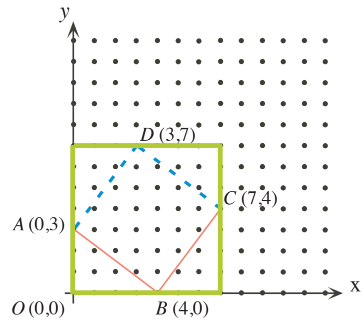


Determining the area of $ABCD$ without using the Pythagorean Theorem

Draw a box with horizontal and vertical sides so that each vertex of the square $ABCD$ is on one of the sides of the box. This creates a large square with sides of length 7 containing four identical triangles and square $ABCD$. Each of the triangles has a base 4 units long and height 3 units long.

$$\begin{aligned}
\text{Area } ABCD &= \text{Area of Large Square} - 4 \times \text{Area of One Triangle} \\
&= \text{Length} \times \text{Width} - 4 \times (\text{Base} \times \text{Height} \div 2) \\
&= 7 \times 7 - 4 \times (4 \times 3 \div 2) \\
&= 49 - 4 \times 6 \\
&= 49 - 24 \\
&= 25 \text{ units}^2
\end{aligned}$$

D is located at (3,7) and the area of the square is 25 units^2. (See the next page for a solution to the area problem using the Pythagorean Theorem.)



Determining the area of $ABCD$ using the Pythagorean Theorem

Since $ABCD$ is a square, it is only necessary to find the length of one side. We can determine the area by squaring the length of the side.

Let the origin be $O(0,0)$. Then OAB forms a right triangle. OA, the distance from the origin to point A on the y-axis, is 3 units. OB, the distance from the origin to point B on the x-axis, is 4 units.

Using the Pythagorean Theorem, we can find AB^2 which is $AB \times AB$, the area of the square.

$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

$\therefore$ **the area of the square is 25 units^2.**