Problem of the Week
Grade 7 and 8

POWERful
Solution

Problem

8^3 means $8 \times 8 \times 8$ and equals 512 when expressed as an integer. When 8^{2011} is expressed as an integer, what is its last digit?

Solution

Let's start by examining the last digit of various powers of 8.

$$\begin{aligned} 8^1 &= \mathbf{8} \\ 8^2 &= 6\mathbf{4} \\ 8^3 &= 51\mathbf{2} \\ 8^4 &= 4\ 09\mathbf{6} \\ 8^5 &= 32\ 76\mathbf{8} \\ 8^6 &= 262\ 14\mathbf{4} \\ 8^7 &= 2\ 097\ 15\mathbf{2} \\ 8^8 &= 16\ 777\ 21\mathbf{6} \end{aligned}$$

Notice that the last digit repeats every four powers of 8. The pattern continues. 8^9 ends with 8, 8^{10} ends with 4, 8^{11} ends with 2, 8^{12} ends with 6, and so on. Starting with the first power of 8, every four consecutive powers of 8 will have the last digit 8, 4, 2, and 6.

We need to determine the number of complete cycles by dividing 2011 by 4.

$$\frac{2011}{4} = 502\frac{3}{4}$$

There are 502 complete cycles and $\frac{3}{4}$ of another cycle. $502 \times 4 = 2008$ so 8^{2008} is the last power of 8 in the 502$^{\text{nd}}$ cycle and therefore ends with 6.

To go $\frac{3}{4}$ of the way into the next cycle tells us that the number 8^{2011} ends with the third number in the pattern, namely 2. In fact, we know that 8^{2009} ends with 8, 8^{2010} ends with 4, 8^{2011} ends with 2, and 8^{2012} ends with 6 because they would be the numbers in the 503$^{\text{rd}}$ complete cycle of the pattern.

Therefore, 8^{2011} ends with the digit 2.