Problem of the Week Grade 7 and 8

Pesky Products Solution

Problem

Three positive numbers exist such that the following is true: the product of the first and second numbers equals the third number; the product of the second and third numbers is 180; and the second number is five times the third number. Determine the product of the three numbers.

Solution

Let the three numbers be represented by a, b, and c.

Since the product of the first and second numbers equals the third number, $a \times b = c$. We are looking for $a \times b \times c = (a \times b) \times c = (c) \times c = c^2$. So when we find c^2 we have found the required product $a \times b \times c$.

We know that $b \times c = 180$ and $b = 5 \times c$ so $b \times c = 180$ becomes $(5 \times c) \times c = 180$ or $5 \times c^2 = 180$. Dividing by 5, we obtain $c^2 = 36$. This is exactly what we are looking for since $a \times b \times c = c^2$.

Therefore, the product of the three numbers is 36.

For those who need to know what the actual numbers are, we can proceed and find the three numbers. We know $c^2 = 36$, so c = 6 since c is a positive number. So $b = 5 \times c = 5 \times (6) = 30$. And finally, $a \times b = c$ so $a \times (30) = 6$. Dividing by 30, we get $a = \frac{6}{30} = \frac{1}{5} = 0.2$. We can verify the product $a \times b \times c = (0.2) \times (30) \times (6) = 6 \times 6 = 36$.

