Grade 9

Number Sense and Numeration: Roots and the Pythagorean Theorem

Play **Square Root Tic-Tac-Toe** first.
Click on http://www.funbrain.com/cgi-bin/ttt.cgi?A1=s&A2=24&A3=0
You can go to www.wiredmath.ca for the links.

The **square root** of a number is one of its *two equal* factors.

For example, $\sqrt{100} = 10$ since $(10) \times (10) = 100$.

The **cube root** of a number is one of its *three equal* factors.

For example, the cube root of 64 is written as $\sqrt[3]{64} = 4$ since $(4) \times (4) \times (4) = 64$.

1. Determine the square root of each number. Do not use a calculator.

a. $\sqrt{100}$ b. $\sqrt{36}$ c. $\sqrt{1}$ d. $\sqrt{81}$ e. $\sqrt{144}$ f. $\sqrt{49}$

g. $\sqrt{169}$ h. $\sqrt{225}$ i. $\sqrt{324}$ j. $\sqrt{900}$ k. $\sqrt{121}$ l. $\sqrt{400}$

m. $\sqrt{\frac{16}{25}}$ n. $\sqrt{\frac{361}{100}}$ o. $\sqrt{\frac{81}{36}}$ p. $\sqrt{\frac{0}{64}}$ q. $\sqrt{\frac{7 \times 7}{5 \times 5}}$ r. $\sqrt{\frac{8 \times 8}{10 \times 10}}$

The quotient property of square roots states:

For any integers a and b where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

For example, $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$.

2. Determine each cube root.

a. $\sqrt[3]{27}$ b. $\sqrt[3]{-125}$ c. $\sqrt[3]{0.008}$ d. $\sqrt[3]{-729}$ e. $\sqrt[3]{\frac{-27}{8}}$ f. $\sqrt[3]{\frac{125}{64}}$

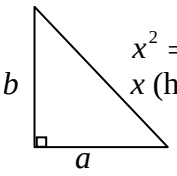
g. $\sqrt[3]{\frac{1}{-216}}$ h. $\sqrt[3]{\frac{512}{1000}}$ i. $\sqrt[3]{\frac{0}{(14)^6}}$ j. $\sqrt[3]{\frac{3^3}{12^3}}$ l. $\frac{\sqrt[3]{-216}}{\sqrt[3]{1728}}$ k. $\sqrt[3]{\frac{9 \times 9 \times 9}{13 \times 13 \times 13}}$

3. a. Estimate each root without a calculator.
   b. Use a calculator to determine each root correct to one decimal place.

i. $\sqrt{10}$ ii. $\sqrt[3]{10}$ iii. $\sqrt{8}$ iv. $\sqrt[3]{-29}$ v. $\sqrt{\frac{7}{26}}$ vi. $\sqrt[3]{-\frac{129}{8}}$

Pythagorean Theorem

The square on the hypotenuse of a right angle triangle is equal to the sum of the squares on the other two sides.

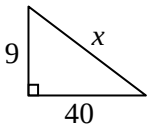


$x^2 = a^2 + b^2$

x (hypotenuse)

Use the Pythagorean Theorem to find an unknown side of a right triangle

To find the length of the hypotenuse:



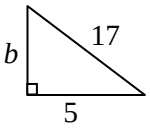
$x^2 = 9^2 + 40^2$

$x^2 = 81 + 1600$

$x = \sqrt{1681}$

$x = 41$

To find the length of a side when the hypotenuse is given:



$5^2 + b^2 = 17^2$

$b^2 = 289 - 25$

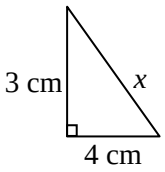
$b^2 = 264$

$b = \sqrt{264}$

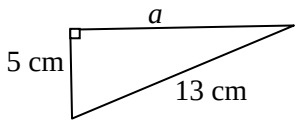
$b \doteq 16.25$ rounded to 2 decimal places.

4. Determine the length of each unknown side.

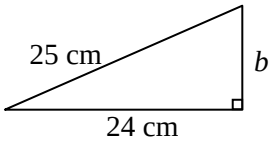
a.



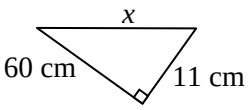
b.



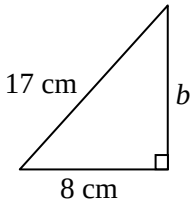
c.



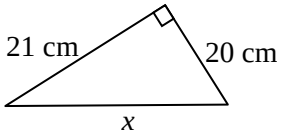
d.



e.



f.



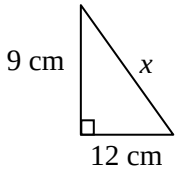
A Slice of History

The Egyptians used sides of 3, 4 and 5 units to obtain a right angle when surveying roads and fields.
They also used it in construction. 3, 4, 5 is the smallest and best known *Pythagorean Triple*.

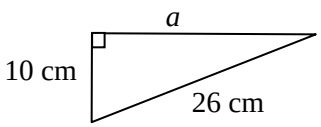
Three integers a, b, and c that satisfy $a^2 + b^2 = c^2$ are called *Pythagorean Triples*.
Other common Pythagorean triples are 6,8,10 (note each side of a 3,4,5 triangle has been multiplied by 2); 5,12,13; 8,15,17; 7,24,25 and 20,21,29.

5. Using Pythagorean triples and multiples determine the length of each unknown side.

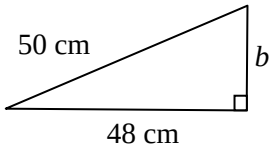
a.



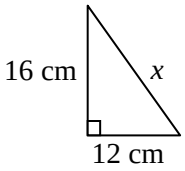
b.



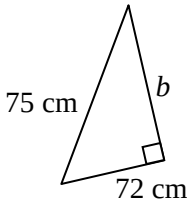
c.



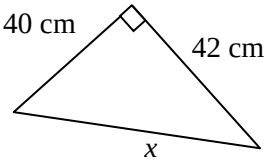
d.



e.

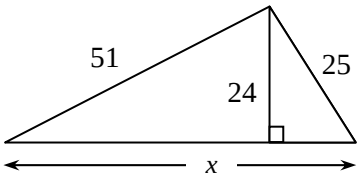


f.

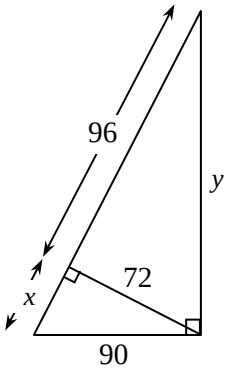


6. Determine the unknown lengths.

a.

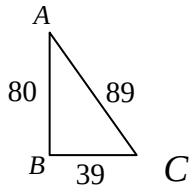


b.



Converse of the Pythagorean Theorem

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite the longest side is a right angle.



E.g. A triangle has sides 39, 80 and 89. Is the triangle right angled?
First determine that $89^2 = 7921$, $80^2 = 6400$ and $39^2 = 1521$.
Since $7921 = 6400 + 1521$, the triangle is right angled at the vertex opposite the longest side of length 89 or at vertex *B* in the diagram.

7. Determine the sets of numbers that can be the lengths of the sides of a right triangle.

a. 15, 20, 25 b. 12, 35, 37 c. 20, 24, 26 d. 1, 1, $\sqrt{2}$

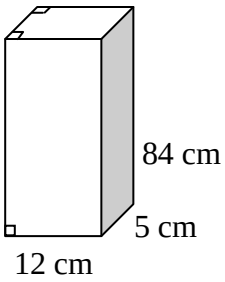
e. 1, $\sqrt{3}$, 2 f. 1.8, 2.4, 3.0 g. 16, 63, 65 h. 3, $\sqrt{34}$, 5

Research

Special Triangles – The lengths of the sides of the triangles given in question 7d and 7e are used frequently in the mathematics. These are the often referred to as $45°, 45°, 90°$ triangle and the $30°, 60°, 90°$ triangles. Use the Internet to find out more about *special triangles*.

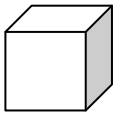
8. Find the length of a diagonal of a rectangular yard 33 m by 56 m.

9. Determine the length of the longest stick that can be placed inside the rectangular prism.



10. The top of a 13 metre wheeled ladder rests against a vertical wall.
    a. The bottom of the ladder rolls away from the base of the wall to a position 5 metres from the wall. How high is the top of the ladder from the base of the wall?
    b. If it rolls again to a position 10 metres from the base of the wall, how much further has the top of the ladder descended?

11. The cube has a total volume of $2744\ \text{cm}^3$.
    a. Determine the length of each edge.
    b. Determine the length of a diagonal of the cube.



12. A spherical balloon has volume $1435\ \text{cm}^3$. The formula $V = \frac{4}{3}\pi r^3$ is used to calculate the volume of a sphere. Determine its radius.

Don't forget now! Go to www.wiredmath.ca for the link.

TRY THESE!

Square Root of a Perfect Square Number

http://www.quia.com/jg/65631.html

Square Root Flashcards

http://www.aplusmath.com/Flashcards/sqrt.html

CHALLENGE YOURSELF!

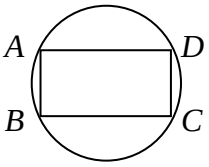
13. Simplify, without using a calculator. Write your answer as a fraction in the form $\frac{a}{b}$, $b \neq 0$.

    a. $\sqrt[3]{\frac{54}{16}}$

    b. $\frac{\sqrt[3]{27+64+125}}{\sqrt{1+8+27+64}}$

    c. $\frac{\sqrt{243}}{\sqrt{75}}$

14. A rectangle is inscribed in a circle. If $AB = 6$ cm and $BC = 8$ cm, determine the area of the circle.



EXTENSION

15. In the expression $S = \sqrt{x_1 + x_2 - x_3 - x_4}$, the variables x_1, x_2, x_3, and x_4 are replaced by 1, 2, 3, and 4 with no repetitions allowed. There are 24 possible replacements. Determine the number of times S will be a real number.