Grade 8

Number Sense and Numeration: Integers - Objects Model

Play **Space Coupe** *http://pbskids.org/cyberchase/games/negativenumbers* and
Circle 99 *http://nlvm.usu.edu/en/nav/grade_g_3.html* first.
You may also go to *www.wiredmath.ca* for the link.

A Slice of History

The + and - symbols first appeared in print in the late 1400's.

They didn't refer to addition or subtraction or to positive or negative numbers, but to surpluses and deficits in business problems.

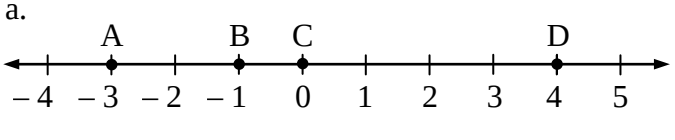
1. Write each statement as an integer.
   a. A withdrawal of $25 from a bank account.
   b. An unexpected gain of $2000 in the stock market.

2. For each sentence, explain what you would do to get back to where you began.
   a. Your climb 85 metres in an aircraft.
   b. You descend 450 metres in a submarine.

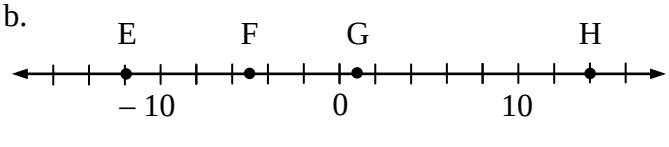
3. Discuss with a partner each of the following statements.
   a. The product of any integer and zero is zero.
   b. The product of any integer and negative one is the same as the opposite of the integer.
   c. One of two integers must be negative, if their quotient is negative.
   d. Both integers must have the same signs, if their quotient is positive.
   e. $10 \div 2 = 5$ since $5 \times 2 = 10$. However, $10 \div 0$ is not defined because there is not an integer which can be multiplied by zero that gives a value of 10.

4. Write a positive or negative integer for each point labelled by the letter.

a.



b.



A _____ B _____ C_____ D_____ E_____ F _____ G _____ H _____

5. Arrange the integers in order from smallest to largest.

   a. $-3, -7, 5, -1$ _____ b. $3, -4, -5, 0, -2$ _____

Let a 'happy face' ☺ be a positive unit (+1) and a 'sad face' ☹ be a negative unit (-1).

Objects Model – Addition
Using faces, $(+4) + (+1)$
can be illustrated as
☺☺☺☺ + ☺
and its result is
☺☺☺☺☺ or the integer +5.

Objects Model – Subtraction
$(+4) - (-1)$ can be illustrated first using 4 faces
as ☺☺☺☺ or
the equivalent of ☺☺☺☺ ☺☹, then remove one sad face ☹ to get
the result of ☺☺☺☺☺ or the integer +5.
Thus, $(+4) - (-1) = 5$.

Multiplication

The product of two integers

Whole Numbers		Integers	
$2+2+2$	and $3+3$	$(+3)+(+3)+(+3)+(+3)$	and $(+4)+(+4)+(+4)$
$=3\times 2$	$=2\times 3$	$=4\times(+3)$	$=3\times(+4)$
$=6$	$=12$	$=12$	$=12$
Since $3\times 2=2\times 3$, we say that multiplication of whole numbers is **commutative**.		The **product** of **two positive integers** is a **positive integer.** **Multiplication** of **integers** is **commutative.**	

6. Write an integer for each product.
   a. $(+8)\times(+3)$ b. $(+9)\times(+4)$ c. $(+6)\times(+1)$ d. $(+3)\times(+8)$ e. $(+4)\times(+9)$

The product of a positive integer and a negative integer

Example.	**Solution.**
Light snow begins to fall. The temperature outside drops $3°C$ every two hours for eight hours. What will be the temperature change in degrees Celsius during the eight hours?	-3 represents the drop in temperature every *two* hours. Temperature change for eight hours will be $(-3)+(-3)+(-3)+(-3)$ $=4(-3)$ repeated addition can be written as a multiplication $=-12$ The temperature change is $-12°C$ after eight hours. **The product of a positive and a negative integer is a negative integer.**

7. Write an integer for each product.
   a. $(+2)\times(-5)$ b. $(+11)\times(-3)$ c. $(-1)\times(+8)$ d. $(-8)\times(+7)$ e. $(-1)\times(+1)$

The product of two negative integers

Consider this pattern.		**Complete these multiplications.**	
$(+4)\times(-5)=-20$ $(+3)\times(-5)=-15$ $(+2)\times(-5)=-10$ $(+1)\times(-5)=-5$ $(0)\times(-5)=$ $(-1)\times(-5)=$ $(-2)\times(-5)=$	As the **multiplier decreases** by $+1$ the **product increases** by $+5$. To continue this pattern suggests the last three products are 0, 5 and 10.	$(-4)\times(+3)=$ $(-4)\times(+2)=$ $(-4)\times(+1)=$ $(-4)\times(0)=$ $(-4)\times(-1)=$ $(-4)\times(-2)=$ $(-4)\times(-3)=$	Each product increases by 4. The products are $-12,-8,-4,0,4,8$ and 12. **The product of two negative integers is a positive integer.** **Also, the product of any integer and zero is zero.**

8. Write an integer for each product.
   a. $(-7)\times(-3)$ b. $(-5)\times(-6)$ c. $(-12)\times(-1)$ d. $(-6)\times(-9)$ e. $(-8)\times(0)$

Division

Division is the inverse of multiplication. $(+5)\times(-4)=-20$. Therefore, $\frac{-20}{+5}=-4$ and $\frac{-20}{-4}=5$.

The **quotient** of **positive and negative integer** is a **negative integer.**
The **quotient** of **two negative integers** is a **positive integer.**

Rules for multiplication of integers

1. The product of two positive integers is a positive integer.
2. The product of two negative integers is a positive integer.
3. The product of a positive integer and a negative integer is a negative integer.

Summary

Same signs	Different signs
$(+)(+)$ $(-)(-)$ $\Big\} = (+)$	$(+)(-)$ $(-)(+)$ $\Big\} = (-)$

Rules for division of integers

The 'rules of signs' for dividing integers are the same as those for multiplying integers.

Summary

Same signs	Different signs
$\frac{(+)}{(+)}$ $\frac{(-)}{(-)}$ $\Big\} = (+)$	$\frac{(+)}{(-)}$ $\frac{(-)}{(+)}$ $\Big\} = (-)$

Helpful analogy

Two players who are on the same team have a positive product. ☺ × ☺ equals a positive ☺ or ☹ × ☹ = ☺. Two players who are on different teams have a negative product. E.g. ☺ × ☹ equals a negative ☹.

9. Write an integer for each quotient. a. $\frac{9}{-3}$ b. $\frac{-35}{7}$ c. $\frac{-72}{-8}$ d. $\frac{-70}{10}$ e. $\frac{66}{-11}$ f. $\frac{-42}{-6}$

10. Using integers, write a mathematical expression to describe each of the following.

a. Five weeks in a row the profit has been \$600.
b. Twenty-eight identical coins are to be shared among 7 brothers and sisters equally.

11. Determine the value of each product.

a. $(+6)\times(+7)$ b. $(+3)\times(-5)$ c. $(-6)\times(+8)$ d. $(-5)\times(-4)$

e. $(+9)\times(+3)$ f. $(+4)\times(-3)$ g. $(-14)\times(+1)$ h. $(-8)\times(-7)$

i. $(0)\times(+7)$ j. $(-10)\times(+10)$ k. $(+11)\times(-11)$ l. $(-7)\times(-7)$

12. Determine the value of each quotient.

a. $(-14)\div(+2)$ b. $(-15)\div(-3)$ c. $(+18)\div(+9)$ d. $(21)\div(-3)$

e. $(-9)\div(+9)$ f. $(-100)\div(-10)$ g. $(+144)\div(+12)$ h. $(63)\div(-9)$

13. Determine the missing integer in each equality.

a. $(+3)\times(\quad)=+18$ b. $(-5)\times(\quad)=+35$ c. $(\quad)\times(-4)=-24$

d. $(+11)\times(\quad)=+121$ e. $(-8)\times(\quad)=-40$ f. $(\quad)\times(-12)=-48$

g. $(+8)\div(\quad)=-2$ h. $(-12)\div(\quad)=+3$ i. $(+35)\div(\quad)=-5$

j. $(\quad)\div(-3)=6$ k. $(\quad)\div(-4)=7$ l. $(-32)\div(-2)=(\quad)$

14. Three students share expenses for a townhouse while at college. Their expense account for October has a balance of $-\$180$. How much does each student owe?

15. Jim loses weight at an average of 3 kg per month. How much does he lose in six months?

16. Christy earned the following gratuities for a week of summer work: Monday $13, Tuesday $14, Wednesday $4, Thursday $26, Friday $18. Determine the average daily gratuity.

17. Calculate the mean of the following profits and losses $-\$10, \$28, -\$16, -\$12, \$6, \$13, -\$37$.

Did You Know?

Dividing a number by 5 is the same as multiplying it by 2 and then dividing the product by 10. Complete a few questions to verify this.

Don't forget now! Go to *www.wiredmath.ca* for the link.

TRY THIS!

Have fun practising your multiplication tables at
http://www.berghuis.co.nz/abiator/tables/frame1.html

INTERESTING RESULT

18. Write any 4-digit number, which has four different digits; for example, 6354. Subtract the smallest number that can be formed using the four digits from the largest number that can be formed from the digits. Repeat this process with the answer until you obtain a surprising result. Compare your result with that of a friend or begin with another 4-digit number and repeat the process. What do you notice?

CHALLENGE YOURSELF!

19. The average of five consecutive integers is 10. What is the sum of the smallest and the largest of these five integers?

EXTENSION

20. At a school play, an elementary school is accepting donations for the Children's Toy Foundation. An average donation of $8 per person is anticipated. The cost of the set design and costumes is $750, advertising in a local newspaper is $125, and posters to announce the concert cost is $89.

a. If 100 people attend the concert, will the school have a profit or loss? Explain.
b. Determine how much profit will be given to the Wish Foundation if 350 attend the concert.
c. Describe a process on how you would determine the number of people who need to attend the concert for the school to make a profit.
d. Follow your process in part c, and determine this minimum number of people.