

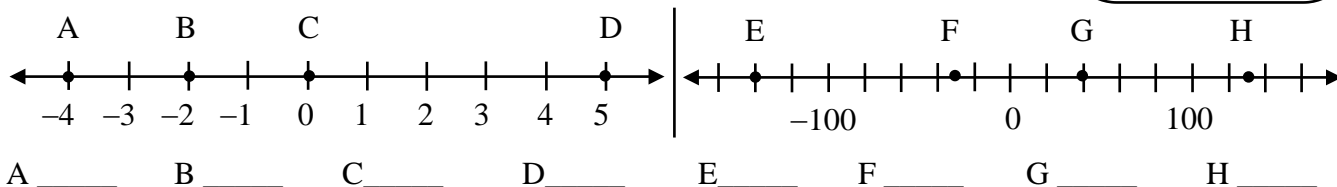
Grade 8

NUMBER SENSE AND NUMERATION: INTEGERS - GRAPHICAL REPRESENTATION

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Play **Space Coupe** <http://pbskids.org/cyberchase/games/negativenumbers> and
Circle 99 http://matti.usu.edu/nlvm/nav/grade_g_3.html first.
You may also go to www.wiredmath.ca for the link.

- Write each statement as an integer.
 - A deposit of \$125 into a bank account.
 - A drop of 4 metres.
- For each sentence, explain what you would do to get back to where you began.
 - You go up twelve floors on an elevator.
 - You drain 5 litres of water from a tub.
- Discuss with a partner each of the following statements.
 - The quotient when zero is divided by any integer except zero is always zero.
 - If two negative integers are multiplied together, their product is a positive integer.
 - The product of a negative integer and a positive integer is a negative integer.
 - The quotient of any positive integer divided by a negative integer has a negative sign.
- Write a positive or negative integer for each point labelled by the letter.



- Arrange the integers in order from smallest to largest.
 - $-1, -6, 4, -3$ _____
 - $3, 0, -2, -6, -4$ _____
 - $-25, 16, -9, 36, -49$ _____

A Slice of History

Mathematicians have used a number of different symbols for multiplication and division throughout history.

Multiplication

The dot (\cdot)
The asterisk ($*$) ($*$)
The cross (\times)

Division

Close parenthesis ($)$
The colon ($:$)
The obelus (\div)

Graphical Representation – Addition

On a graph $(+4) + (+1)$
can be illustrated with arrows AB and CD as

and its result is arrow EF
or the integer +5. Thus, $(+4) + (+1) = 5$.

Graphical Representation – Subtraction

On a graph $(+4) - (+1)$
can be illustrated with arrows GH and IJ

and its result is arrow KL
or the integer +3. Thus, $(+4) - (+1) = 3$.

Multiplication

The product of two integers

Whole Numbers	Integers
$3 + 3 + 3 + 3$ and $4 + 4 + 4$ $= 4 \times 3$ $= 3 \times 4$ $= 12$ $= 12$ Since $4 \times 3 = 3 \times 4$, we say that multiplication of whole numbers is commutative .	$(+5) + (+5) + (+5) + (+5)$ and $(+4) + (+4) + (+4) + (+4) + (+4)$ $= 4 \times (+5)$ $= 5 \times (+4)$ $= 20$ $= 20$ The product of two positive integers is a positive integer. Multiplication of integers is commutative.

6. Write an integer for each product.
 a. $(+4) \times (+7)$ b. $(+6) \times (+3)$ c. $(+1) \times (+4)$ d. $(+7) \times (+4)$ e. $(+3) \times (+6)$

The product of a positive integer and a negative integer

Example. Mohammad decides to follow an exercise program designed by his trainer. He anticipates that his program will lead to a weight loss of 2 kg per month for the first four months. What will be Mohammad's total change in weight in four months?	Solution. -2 represents the loss of weight each month. Weight change for four months will be $(-2) + (-2) + (-2) + (-2)$ $= 4(-2)$ repeated addition can be written as a multiplication $= -8$ Mohamad's weight change is -8 kg after four months. The product of a positive and a negative integer is a negative integer.
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7. Write an integer for each product.
 a. $(+3) \times (-7)$ b. $(+5) \times (-8)$ c. $(-1) \times (+4)$ d. $(-9) \times (+5)$ e. $(+1) \times (-1)$

The product of two negative integers

Consider this pattern. $(+4) \times (-3) = -12$ $(+3) \times (-3) = -9$ $(+2) \times (-3) = -6$ $(+1) \times (-3) = -3$ $(0) \times (-3) =$ $(-1) \times (-3) =$ $(-2) \times (-3) =$ $(-3) \times (-3) =$	As the multiplier decreases $y + 1$ the product increases $y + 3$. To continue this pattern suggests the last three products are 0, 3 and 6.	Complete these multiplications. $(-2) \times (+3) =$ $(-2) \times (+2) =$ $(-2) \times (+1) =$ $(-2) \times (0) =$ $(-2) \times (-1) =$ $(-2) \times (-2) =$ $(-2) \times (-3) =$	Each product increases by 2. The products are $-6, -4, -2, 0, 2, 4$ and 6. The product of two negative integers is a positive integer. Also, the product of any integer and zero is zero.
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8. Write an integer for each product.
 a. $(-5) \times (-6)$ b. $(-4) \times (-8)$ c. $(-1) \times (-7)$ d. $(-9) \times (-8)$ e. $(-1) \times (0)$

Division

Division is the inverse of multiplication. $(+3) \times (-5) = -15$. Therefore, $\frac{-15}{+3} = -5$ and $\frac{-15}{-5} = 3$.

The **quotient of positive and negative integer is a negative integer.**

The **quotient of two negative integers is a positive integer.**

9. Write an integer for each quotient. a. $\frac{-10}{5}$ b. $\frac{12}{-4}$ c. $\frac{-24}{-6}$ d. $\frac{-14}{2}$ e. $\frac{22}{-11}$ f. $\frac{-27}{-9}$

Rules for multiplication of integers

1. The product of two positive integers is a positive integer.
2. The product of two negative integers is a positive integer.
3. The product of a positive integer and a negative integer is a negative integer.

Summary

Same signs

$$\begin{array}{l} (+)(+) \\ (-)(-) \end{array} \left. \vphantom{\begin{array}{l} (+)(+) \\ (-)(-) \end{array}} \right\} = (+)$$

Different signs

$$\begin{array}{l} (+)(-) \\ (-)(+) \end{array} \left. \vphantom{\begin{array}{l} (+)(-) \\ (-)(+) \end{array}} \right\} = (-)$$

Rules for division of integers

The 'rules of signs' for dividing integers are the same as those for multiplying integers

Summary

Same signs

$$\begin{array}{l} \frac{(+)}{(+)} \\ \frac{(+)}{(-)} \\ \frac{(-)}{(-)} \\ \frac{(-)}{(+)} \end{array} \left. \vphantom{\begin{array}{l} \frac{(+)}{(+)} \\ \frac{(+)}{(-)} \\ \frac{(-)}{(-)} \\ \frac{(-)}{(+)} \end{array}} \right\} = (+)$$

Different signs

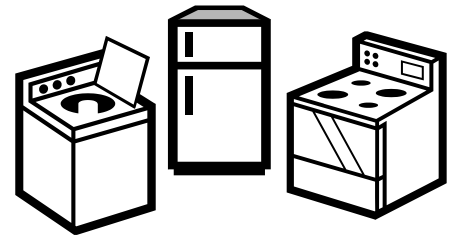
$$\begin{array}{l} \frac{(+)}{(-)} \\ \frac{(-)}{(+)} \\ \frac{(-)}{(-)} \\ \frac{(+)}{(+)} \end{array} \left. \vphantom{\begin{array}{l} \frac{(+)}{(-)} \\ \frac{(-)}{(+)} \\ \frac{(-)}{(-)} \\ \frac{(+)}{(+)} \end{array}} \right\} = (-)$$

Helpful analogy

Two players who are on the same team have a positive product. $\odot \times \odot$ equals a positive \odot or $\ominus \times \ominus = \odot$.
Two players who are on different teams have a negative product. E.g. $\odot \times \ominus$ equals a negative \ominus

10. Using integers, write a mathematical sentence describing each of the following.
- a. The temperature is dropping 2°C per hour for eight hours.
 - b. An eight-piece pizza is to be shared by four students.
11. Determine the value of each product.
- a. $(+10) \times (+3)$
 - b. $(+4) \times (-7)$
 - c. $(-8) \times (+7)$
 - d. $(-9) \times (-6)$
 - e. $(+8) \times (+4)$
 - f. $(+3) \times (-11)$
 - g. $(-12) \times (+1)$
 - h. $(-5) \times (-5)$
 - i. $(+1) \times (+7)$
 - j. $(-9) \times (+8)$
 - k. $(+12) \times (-12)$
 - l. $(-6) \times (-6)$
12. Determine the value of each quotient.
- a. $(-48) \div (+6)$
 - b. $(-63) \div (-7)$
 - c. $(+24) \div (+6)$
 - d. $(35) \div (-7)$
 - e. $(-11) \div (+11)$
 - f. $(-81) \div (-9)$
 - g. $(+72) \div (+12)$
 - h. $(49) \div (-7)$
13. Determine the missing integer for each equality.
- a. $(+7) \times () = +28$
 - b. $(-3) \times () = +36$
 - c. $() \times (-9) = -63$
 - d. $(+12) \times () = +84$
 - e. $(-7) \times () = -56$
 - f. $() \times (-4) = -48$
 - g. $(+36) \div () = -6$
 - h. $(-72) \div () = +8$
 - i. $(+42) \div () = -6$
 - j. $() \div (-2) = 6$
 - k. $() \div (-4) = 5$
 - l. $(-44) \div (-4) = ()$

14. A storeowner pays \$30 per square metre per year for rent. The store has an area of 1200 square metres. What is his yearly rent?
15. An appliance-store owner had daily receipts as follows: Monday \$480, Tuesday \$975, Wednesday \$809, Thursday \$727, Friday \$1043, Saturday \$2980. What were the average daily receipts?
16. Calculate the mean of the following profits and losses
\$15, -\$36, \$47, -\$11, -\$64, \$37.
17. What number when divided by 12 gives a quotient of 16 and a remainder of 8?



Did You Know?

Divisibility by 13. Delete the last digit of the number and then subtract 9 times the deleted digit. If the remaining number is divisible by 13 then so is the original number.



Don't forget now! Go to www.wiredmath.ca for the link.

TRY THESE!

Have fun practicing your multiplication tables at
<http://www.berghuis.co.nz/abiator/tables/frame1.html>

CHALLENGE YOURSELF!

18. What number when divided by 83 gives a quotient of -37 and a remainder of 23?
19. The present temperature is 16°C . A cold front enters the atmosphere causing the temperature to drop an average of 3°C per hour for 8 hours. Explain whether or not any precipitation that falls after 8 hours will be snow or rain. What increase or decrease in temperature is necessary to bring the temperature to the freezing point?
20. Aircraft A departed from an airport at 8:00 flying at 400 km/h. Aircraft B departed from the same airport at 11:00 flying at 550 km/h, on the same course as A. How far apart were they at 17:00?



EXTENSIONS

21. Absolute value is the size, or magnitude, of a number x with or without the negative sign. For example, the absolute value of 8 or of -8 is 8. We write absolute value using vertical lines so the "absolute value of x " is denoted $|x|$.
- a. Determine each of the following.
- i. $|-13|$ ii. $|0|$ iii. $|5|$ iv. $|-18| - |-7|$ v. $|-9| - 3|4| - 2|-4|$
- b. Determine the number of integral solutions of $|x| \times |y| = 20$.