

Grade 7

NUMBER SENSE AND NUMERATION: MULTIPLES, FACTORS AND SQUARE ROOTS

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Play **The Factor Game** http://illuminations.nctm.org/index_d.aspx?id=433 and **Multiplication Mystery** <http://www.hbschool.com/activity/mult/mult.html> first! You may go to www.wiredmath.ca for the link.

8, 12 and 28 are **multiples** of 4 because
 $8 = 4 \times 2$, $12 = 4 \times 3$ and $28 = 4 \times 7$.

1. a. Complete the table.

	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 3	3											
Multiples of 7	7											

- b. Which numbers from the chart are multiples of both 3 and 7? _____

2. a. Complete the table.

	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 4	4											
Multiples of 6	6											

- b. Which numbers are multiples of both 4 and 6? _____
c. What number is the least common multiple (LCM) of 4 and 6? _____

3. a. Write the smallest twelve positive numbers that are multiples of both 2 and 6.
b. What is the least common multiple of (LCM) of 2 and 6?

4. Determine the least common multiple of these pairs of numbers.

- a. 3, 4 b. 5, 6 c. 6, 8 d. 2, 14 e. 4, 10 f. 9, 12

Divisibility Rules for Integers

- 2 – A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8; i.e., if the number is even.
- 3 – A number is divisible by 3 if the sum of its digits is divisible by 3.
E.g. 42 is divisible by 3 because $4 + 2 = 6$ and 6 is divisible by 3
- 4 – A number is divisible by 4 if the number formed by its last two digits is divisible by 4.
- 5 – A number is divisible by 5 if its last digit is 0 or 5.
- 6 – A number is divisible by 6 if it is divisible by 2 and by 3.
- 8 – A number is divisible by 8 if the number formed by its last three digits is divisible by 8.
- 9 – A number is divisible by 9 if the sum of its digits is divisible by 9.
- 10 – A number is divisible by 10 if its last digit is 0.

5. Use the divisibility rules to determine which number is not a multiple of the first number.
- a. 3; 23, 9, 18, 42 b. 5; 45, 30, 62, 75 c. 4; 28, 116, 196, 154
d. 3; 48, 72, 66, 88 e. 9; 54, 117, 63, 134 f. 8; 96, 120, 192, 178

Did You Know?

Traffic lights were used before motorcars became popular. In 1868, a lantern with red and green signals was used at an intersection in London, England to control the flow of horse buggies and pedestrians.

6. A car dealership has 84 new vehicles. Can the new vehicles be arranged in three rows with the same number of vehicles in each row?
7. The month of January has 31 days. On Saturday January 1, 2005, Mao began an exercise program by lifting weights and running three kilometres. Mao decided to weight train every second day and run every third day.
- On which day of the week and on what date in January will he next both lift weights and run?
 - How many times in January will he both lift weights and run on the same day?
 - How many kilometres will he run in January?
 - If Mao continues to run every third day, how many times would he run in 2005?
8. Buns are sold by the dozen (12) and hot dogs come in packages of 8. What is the fewest number of each required so that there are no buns or hot dogs left over after lunch?

A **factor** is a natural number that divides exactly into another number; that is, without a remainder.

» For example, the factors of 10 are 1, 2, 5, and 10.

A **prime number** is a natural number greater than 1 that has only two factors, itself and 1.

» For example, 29 has two factors 1 and 29.

A **composite number** is a natural number that has factors other than itself and 1.

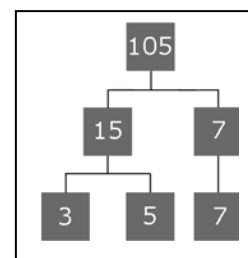
» For example, 8 has four factors 1, 2, 4, 8.

Prime factorization is an expression showing a composite number as a product of its prime numbers.

» For example, the prime factorization of 105 is $3 \times 5 \times 7$.

9. a. List all the factors of each number.
i. 9 ii. 17 iii. 16 iv. 36 v. 43 vi. 100
- b. Which of the numbers in question 9a are prime and which are composite?
10. Write the prime factorization of each number.
- a. 14 b. 110 c. 123
d. 36 e. 350 f. 1000

Factor Tree



11. a. Write the prime factorization of each number in the pair.
 b. Determine the greatest common factor (GCF) of each pair.
- i. 10 and 15 ii. 12 and 20 iii. 18 and 24 iv. 21 and 35 v. 54 and 72

A Slice of History

The ancient mathematician Archimedes stated that $\sqrt{3}$ is greater than $\frac{265}{153}$ but less than $\frac{1351}{780}$. To this day, mathematicians do not know how Archimedes arrived at this result.

The **square root** of a perfect square is one of the two *equal* factors.
 For example, the square root of 25 is written as $\sqrt{25} = 5$ since $(5) \times (5) = 25$.

12. Determine the square root of each number.

- a. $\sqrt{4}$ b. $\sqrt{9}$ c. $\sqrt{1}$ d. $\sqrt{16}$ e. $\sqrt{0}$

The symbol $\sqrt{\quad}$ is called a radical sign.

13. Complete the chart.

Whole number	0	1	4	9	16		36		64	81			144
Square root						5		7			10	11	

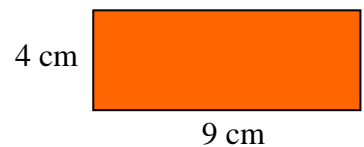
Squares and square roots occur in applications. It is an advantage to memorize the squares and square roots in this chart.

14. Determine these square roots. Use a calculator only as a last resort.

- a. $\sqrt{400}$ b. $\sqrt{225}$ c. $\sqrt{324}$ d. $\sqrt{256}$ e. $\sqrt{169}$
 f. $\sqrt{361}$ g. $\sqrt{196}$ h. $\sqrt{576}$ i. $\sqrt{900}$ j. $\sqrt[3]{625}$

15. Determine the length of a side a square that has an area of 25 cm².
16. A square has an area of 36 cm². What is the length of each side of the square?

17. a. Determine the area of the rectangle at the right.
 b. If this rectangle were a square with the same area, what would be the length of each side?



18. Two sides of a rectangle are given.
- a. Determine the area of each rectangle.
 b. Determine the length of one side of a square with the same area as the rectangle.
- i. 4 cm and 16 cm ii. 5 cm and 45 cm iii. 7 m and 28 m

Don't forget now! Go to www.wiredmath.ca for the link.



TRY THESE!

Find the square root of a perfect square.

<http://www.quia.com/jg/65631.html>

Determine the value of square roots on the flash cards

<http://www.aplusmath.com/Flashcards/sqrt.html>

CHALLENGE YOURSELF!

19. Determine each value.

a. $\sqrt{13 + \sqrt{8 + \sqrt{1}}}$

b. $(\sqrt{225} - \sqrt{144})^2$

20. The product of the ages of two persons over the age of 19 is 770. Determine the sum of their ages.



EXTENSIONS

21. Determine the least positive integer by which each should be multiplied so that the product is a perfect square.

a. 28

b. 126

22. Divisibility by 7: Take the units digit and multiply it by 2. Subtract the result from the number shortened after removing the unit's digit of the original number. Determine that this result is divisible by 7. Use this rule over and over again as necessary. For example, 336 is divisible by 7 because the units digit of 6 times 2 equals 12. Then 33 subtract 12 equals 21. We know 21 is divisible by 7.

Which of these numbers 973, 60 494, 5 885, and 12 345 676 is divisible by 7?

23. List all two-digit positive integers with the following property. Both the original number and the different new two-digit number obtained by interchanging the digits has a common factor other than 1.