Grade 9

Number Sense And Numeration: Algebra and Polynomials

Play the **Algebra Millionaire Game** first.
Click on http://www.quia.com/rr/4096.html to play.
You may also go to www.wiredmath.ca for the link.

- A **term** is a number, or a variable, or the product of numbers and variables.
  For example, the expression $x^2 + 7x$ has two terms: x^2 and $7x$.
- A **coefficient** is the numerical factor of the term.
  For example, 5 is the coefficient in the term $5x^2y$.
- A **constant** does not include a variable.
  For example, 8 is the constant in $x^3 + 8$.
- **Like terms** have the same variables and are raised to the same exponent.
  For example, $6x^3$ and $-4x^3$ are like terms.
- **Unlike terms** have different variables or the same variable raised to a different exponent.
  For example, $2x$, $3y$ and $4z$ are unlike terms, as well as $7x^2$ and $-9x^3$.

1. Complete the following chart.

Expression	Number of Terms	Coefficient of x	Constant
$5x$			
$2x+4y$			
$3x^2-6x+4$			
$2x+16$			
$4z^3+3y^2-5x-10$			

Fun Fact!

Many mathematical terms are derived from Greek words. The terms **monomial**, **binomial** and **trinomial** come from the prefixes "mono", "bi" and "tri" meaning 1, 2 and 3. A monomial has 1 term, a binomial has 2 terms and a trinomial has 3 terms.

2. Circle the like terms.

a. $3y, 2x, -3x$

b. $-4g,\ 3h,\ -2g$

c. $-6x,\ 2x^2,\ -4x^2,\ 4$

d. $xy,\ 4x^2y,\ 3xy^2,\ 2yx$

e. $6lmn,\ 4lm^3n,\ 5n^2m^2, (3mn)^2$

f. $0.3m,\ 2.5n^3,\ 2n,\ 3.2n^3,\ \frac{1}{2}n^3$

3. Simplify.

a. $3x-2x+4x$

b. $-y-y+3y$

c. $12x^2+2x^2-3^2$

d. $5s^4-4s^4+3s^2+2s^2$

e. $2n-3-4n+8$

f. $-4x+2-7+7x$

g. $0.4x-0.1x+0.2y+x$

h. $-x^3-2x^2+3x^2+5x^3$

i. $(4h+5)-(2h+3)$

j. $(2x-3y)+(3x+y)$

k. $(2x^2-2x+3)+(-x^2+3x-2)$

l. $(3x^2-4xy+6y^2)-(6x^2-8xy-3y^2)$

Distributive Law

In algebra, when we multiply a polynomial by a monomial, we use the distributive law to multiply every term in the polynomial by the monomial:

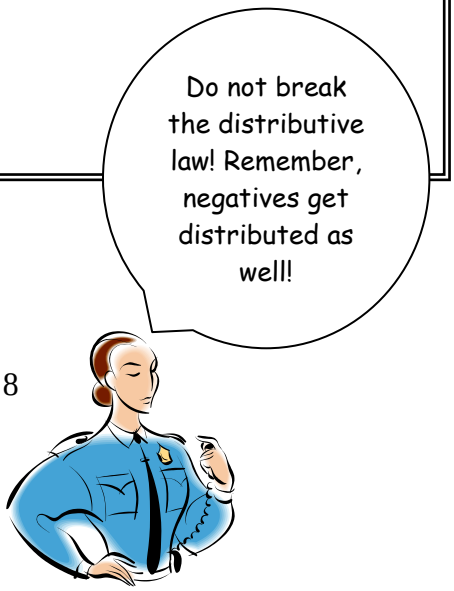
$a(b+c) = ab + ac$, $a(b-c) = ab - ac$

For example: $-3x(5x-4)+8x$

$= -15x^2 + 12x + 8x$

$= -15x^2 + 20x$

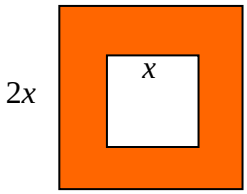
$= -5x(3x-4)$



4. Expand and simplify where necessary.
   a. $5(a+2)$
   b. $-4(2-3x)$
   c. $2(x^2 - 4x + 1)$
   d. $2(3x-1) - 5(4x+2)$
   e. $4(5x-1) - (2x-3)$
   f. $\frac{3}{2}\left(\frac{1}{3}a - \frac{2}{3}b\right) - \frac{3}{4}\left(\frac{1}{3}a + \frac{2}{3}b\right) + 8$

5. Multiply the polynomial by the monomial.
   a. $(8x+4)(7y)$
   b. $(5x-3)(2x)$
   c. $(5x^2 + 3x + 7)(19xy)$
   d. $(5y + 7x)(-3xy^2)$
   e. $(-4x)(3x^2 - 2xy + y^2)$
   f. $(3x^4y^3 + 2x^3y^3 + 2xy^2)(-xy^4)$

6. Write an equation to represent the area of:
   a. the large square.
   b. the small square.
   c. the shaded region.



7. John has change in his pocket consisting of quarters, loonies ($1 coins, the one on the left) and toonies ($2 coins, the one on the right). The number of quarters is twice the number of loonies and the number of loonies is twice the number of toonies.
   a. Write an algebraic expression to represent the number of coins he has, using only one variable.
   b. Write an algebraic expression to represent the value of these coins.
   c. How much money does he have in his pocket, if he has two toonies?

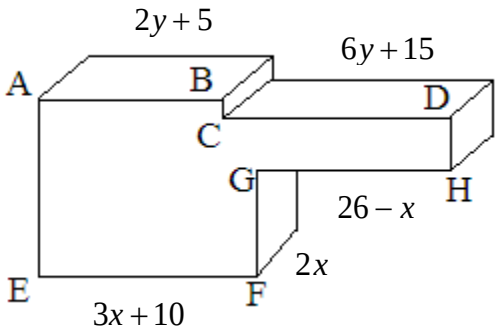
Fun Fact!

The inner portion of the toonie is made primarily out of copper while the outer ring is made mostly out of nickel. If the diameter of the coin is about 28 mm and the diameter of the inner portion is 21 mm, can you write an equation modeling the area of the inner portion on a toonie?

8. Michelle parks her car in a private parking lot for $1.50 per hour plus an initial parking fee. She paid $12.50 for 6 hours of parking.
   a. What was the initial parking fee?
   b. Write a formula for the cost of parking?
   c. How much money does she need to pay if her car was parked for 8 hours?

9. Solve each equation.
   a. $5k+3=8$
   b. $4+7m=4m-2$
   c. $15-3z=4z+8$
   d. $2(x-3)-5=6$
   e. $-14b-5=12b+8$
   f. $5(x+4)-10=5+4(x-1)$
   g. $14x-16=6+2x+2$
   h. $3(a-2)+5a=14-6(5-3a)$

10. A machine part has the dimensions as shown. (All the dimensions are in centimetres.)



   a. If $EF = GH$, calculate the value of x.
   b. What is the total width ($EF + GH$) of the part?
   c. How are the lengths $AB + CD$ and $EF + GH$ related?
   d. Determine the value of y.

Tips for solving equations

1. Identify the variable.
2. Expand and simplify the equations.
3. Arrange the equation so that all variables are on the same side and all numbers are on the opposite side.
4. Solve for the variable by doing arithmetic operations.

11. Find three consecutive numbers with the sum 144.

12. Jack is preparing a birthday gift for his nephew Charlie and bought the number of candles that corresponds to his age. The sum of Jack's and Charlie's ages equals 30. In 5 years, Jack will be three times older than Charlie. How many candles should Jack add to the cake on the right? How old is Jack now?

13. The rental costs, C, of two car agencies are based on distance driven in kilometres, d, and a basic fee. For a car rental, car agency A charges $1.25 per kilometre plus $30.00 of basic fee and car agency B charges $1.50 per kilometre plus $20.00 of basic fee.
   a. Write an equation that represents the rental costs charged by each of two car agencies.
   b. Mary plans to rent a car for a day and intends to drive 30 km. From which company should she rent a car to minimize cost?
   c. William also plans to rent a car for a day trip of 150 km. From which company should he rent a car to minimize the cost?
   d. Calculate the distance for which the rental costs of the two companies are the same.
   e. What is the cost of driving this distance?

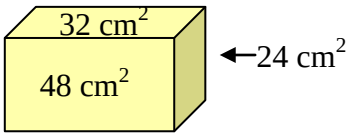

Company A


Company B

CHALLENGE YOURSELF!

14. In a group of men and women, the average age is 31. If the men's ages average 35 years, and the women's ages average 25. What is the ratio of the number of men to the number of women?

15. The side, front, and bottom faces of a rectangular solid have areas of 32, 24 and 48 cm^2 respectively. What is the volume of the rectangular solid, in cm^3?



EXTENSIONS

Binomial Expansion

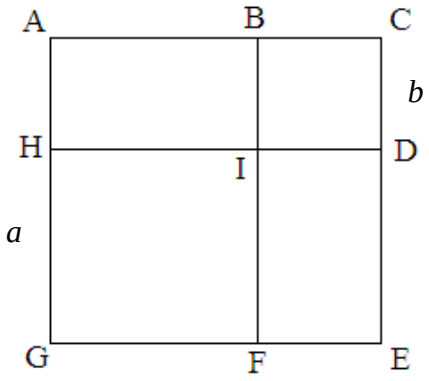
$$(a+b)^2 = (a^2 + 2ab + b^2) \text{ or } (a-b)^2 = (a^2 - 2ab + b^2)$$

Proof: using distributive law

$$(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

Similarly, $(a-b)^2 = a^2 - 2ab + b^2$.

16. The square ACEG is made of two squares and two rectangles.
If $HG = a$ and $CD = b$,
    a. What is the area of the square HIFG?
    b. What is the area of the square BCDI?
    c. What is the area of the rectangle ABIH?
    d. What is the area of the rectangle IDEF?
    e. What is the area of the square ACEG?
    f. What is the relationship between the area of the square ACEG and the sum of the squares HIFG, BCDI and the rectangles ABIH and ACEG?



17. Use the formula to expand the following expressions.
    a. $(3a+2b)^2$
    b. $(2x-y)^2$
    c. $\left(\frac{1}{2}m + \frac{1}{3}n^2\right)^2$
    d. $(a+b+c)^2$

18. Use your knowledge of $(a+b)^2$ and your result in 17d) to write down the expansion of $(a+b+c+d)^2$.

Try These!
Resolve Equations
http://www.quia.com/jq/10288.html