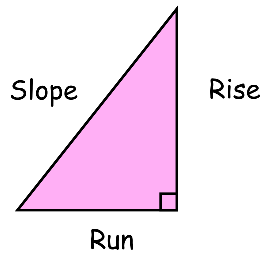
Grade 9

Linear Relations: Slopes and the Equation of a Line

Play these **Slope Games** *http://www.quia.com/jg/63508.html* first.
You may also go to *www.wiredmath.ca* for the link.

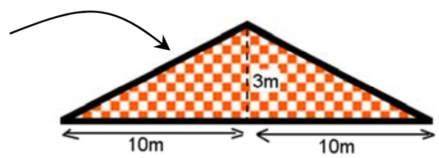
The **slope** of a line is the measure of the amount of steepness of the line or the pitch of a roof.
The slope of a line, *m*, is the quotient of the vertical change (rise) divided by the horizontal change (run).

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} \quad \text{i.e.} \quad m = \frac{\text{rise}}{\text{run}} \quad \text{i.e.}$$

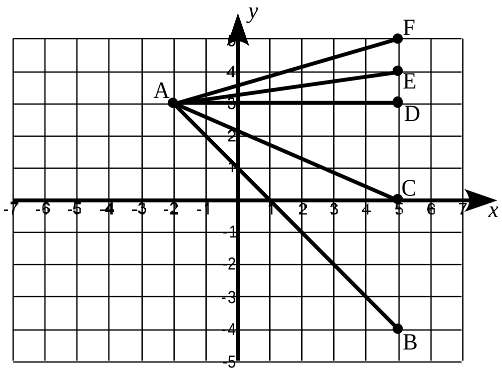


E.g. Determine the slope of this roof.

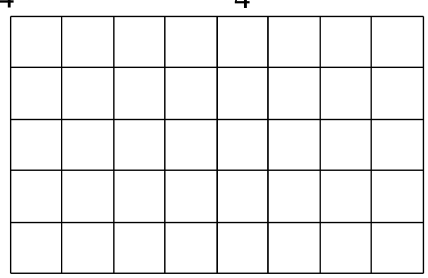
Solution: $m = \frac{\text{rise}}{\text{run}} = \frac{3}{10}$



1. Find the slope of each line segment.



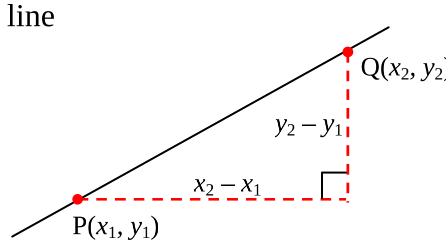
2. Using the grid below, draw line segments with slopes of $\frac{1}{4}$, 4, -4, and $-\frac{1}{4}$.



Given two points P(x_1, y_1) and Q(x_2, y_2) on a line, the slope of the line may be found using:

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_2 \neq x_1$$

- Horizontal lines have a slope of 0.
- Vertical lines have no slope.



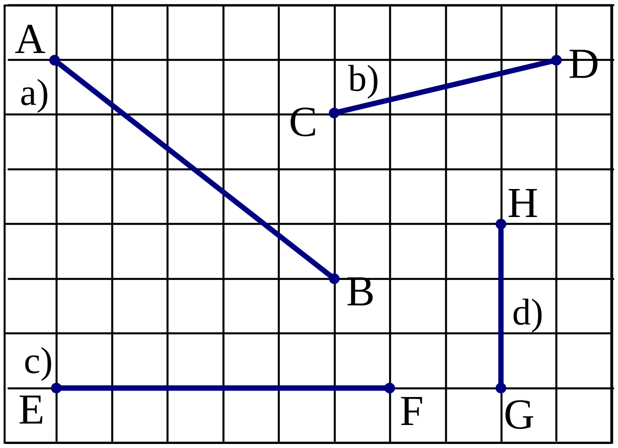
E.g. Find the slope of the line joining P(-3, 4) and Q(-1, -2).

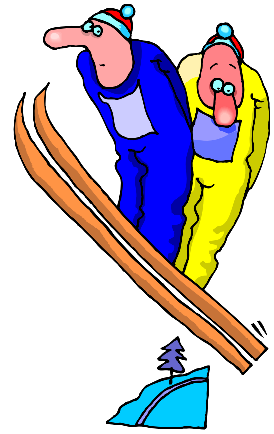
Solution: $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad (x_1, y_1) = (-3, 4)$, $(x_2, y_2) = (-1, -2)$

Therefore, $m_{PQ} = \frac{-2-4}{-1-(-3)} = \frac{-6}{2} = -3.$

Expectations: i) determine the slopes of a line segment or a line. ii) express the equation of a line in the form $y = mx + b$. *For more activities and resources from the University of Waterloo's Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.*

3. Determine the slopes of the line segments below?





4. Are the slopes of these lines negative, positive, or 0?

a.

b.

c.

d.

5. Determine the slope of the line that passes through each pair of points?

a. A(0, 3), B(4, 3)

b. R(12, 10), S(12, 3)

c. G(2, 5), H(4, 11)

d. L(9, 5), M(8, 9)

e. X(8, 3), Y(11, -9)

f. U$(-3, -4)$, V(3, 4)

6. Given the slope of the line segment and the other end point, determine the unknown.

a. (0, 5), (a, 1), slope 2

b. (b, 0), (10, 9), slope $\frac{3}{2}$

c. (17, 21), (c, 5), slope -8

d. (7, 12), (12, d), slope 6

e. (17, e), (32, 9), slope $\frac{7}{5}$

f. (3, f), (7, 4), slope $-\frac{1}{2}$

To determine the equation of a line when two points on the line are given.

1. Determine the slope.
2. Determine the value of b by substituting the coordinates from a point on the line and the slope into $y = mx + b$.
3. Write the equation, using the values for m and b found above.

E.g. Determine the equation of the line that passes through points $A(5, 2)$ and $B(-1, 8)$.

1. $m_{AB} = \frac{8-2}{-1-(5)} = \frac{6}{-6} = -1$
2. $y = mx + b$, and using the point $A(5, 2)$ that lies on the line
   $2 = -1(5) + b$
   $2 = -5 + b$
   $b = 7$
3. Therefore, the equation of the line is $y = -x + 7$.

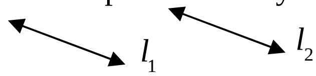
Expectations: i) determine the slopes of a line segment or a line. ii) express the equation of a line in the form $y = mx + b$. *For more activities and resources from the University of Waterloo's Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.*

7. The coordinates of two points are given. Determine the equation of the line.

a. $(0, 0)$ and $(-2, 8)$ b. $(4, 2)$ and $(8, -1)$ c. $(10, 21)$ and $(-10, -19)$

d. $(2, -17)$ and $(-9, 60)$ e. $(-6, 25)$ and $(4, 5)$ f. $(-8, -1)$ and $(-9, 2)$

Parallel Lines: two lines are parallel if they have the same slope $m_1 = m_2$



8. Determine an equation of a line that passes through $P(9, 6)$ and is parallel to

a. $y = 3x + 4$ b. $y = -\frac{7}{4}x$ c. $y = \frac{4}{3}x - 12$

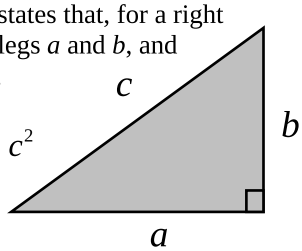
CHALLENGE YOURSELF!

Pythagorean Theorem

Pythagoras, an ancient mathematician, discovered this important theorem in the sixth century B.C.

The theorem states that, for a right triangle with legs *a* and *b*, and hypotenuse *c*,

$$a^2 + b^2 = c^2$$



the area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the other two sides.

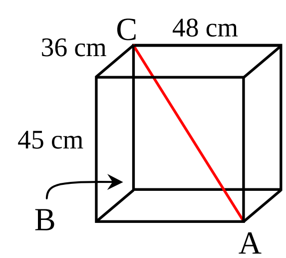
For practice on the Pythagorean theorem, visit http://www.gomath.com/algebra/pythworksheet.php .

9. Two ladders, each 17 metres in length, are leaning against a wall. The foot of one of the ladders is 15 metres away from the base of the wall. The other ladder has its foot 8 metres away from the same base of the same wall. How far up the wall will the ladder with the greater slope reach?

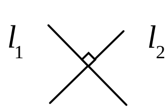
10. The inside dimensions of a crate are $36\text{ cm} \times 48\text{ cm} \times 45\text{ cm}$, as shown.

a. Determine the length of the diagonal, AB, of the base.

b. What is the length of the longest item that can be placed in the crate? (line AC)



Perpendicular Lines: two lines are perpendicular if the product of their slopes is -1 (that is, $m_1 \times m_2 = -1$). We say the slopes are <u>negative reciprocals</u>.



$$m_1 \times m_2 = -1$$

11. Determine the slope of a line segment perpendicular to a line segment with each given slope.

a. $m = 2$ b. $m = -4$ c. $m = \frac{1}{3}$

d. $m = -\frac{2}{7}$ e. $m = \frac{9}{7}$ f. $m = 0$

12. Determine an equation of a line that passes through the point $P(9,6)$ and is perpendicular to

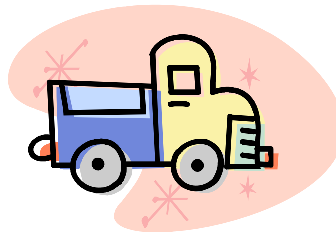
a. $y = 3x + 4$ b. $y = -\frac{7}{4}x$ c. $y = \frac{4}{3}x - 12$

13. Two perpendicular lines, labelled A and B, are drawn on a grid. The slope of line A is 4 and it intersects the other line at the point $(3,-5)$.

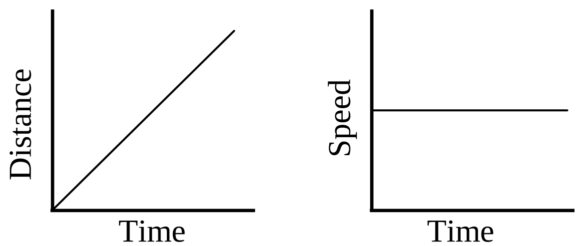
a. What is the equation of line A?
b. What is the equation of line B?

EXTENSIONS

14. Two trucks left two towns A and B at the same time, and each was headed to the other town at a constant speed, passing each other at point C. The truck from town B completed the journey from C to A in 20 minutes. The truck from A completed the journey from A to C in 45 minutes, while maintaining its steady speed of 45 km/h. Find the speed of the truck from B in km/h, assuming that it maintains a constant speed throughout.



15. The following are distance-time and speed-time graphs that depict the same constant motion. What can be said of the relationship between the slope of the distance-time graph and the speed of the speed-time graph? Why do you think this is so?



Did You Know?

Mount Everest, the world's highest peak at approximately 8850 m high, has an average slope of $\frac{177}{320}$ m, because it has an average radius of 16 km!