## Grade 9

## Linear Relations: Graphing and analyzing

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## Different Forms of Linear Function

Please go to http://id.mind.net/~zona/mmts/functionInstitute/linearFunctions/linearFunctions.html first! You may also go to www.wiredmath.ca for the link.

A table of values represent a linear relationship if:

- a straight line is formed when all the points are connected together.
- the finite differences are all the same for every row in the difference table.

1. State if each graph represents a linear or nonlinear relationship?
a.

b.

c.

d.

e.

f.

2. Create a difference table for each set of data. Does the data represent a linear or nonlinear relationship?
a.

| $x$ | $y$ |
| :---: | :---: |
| 2 | 4 |
| 3 | 9 |
| 4 | 14 |
| 5 | 19 |
| 6 | 24 |

b.

| $x$ | $y$ |
| :---: | :---: |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |


| $x$ | $y$ |
| :---: | :---: |
| 5 | -3 |
| 4 | -2 |
| 3 | -1 |
| 2 | 0 |
| 1 | 1 |

See how a line of best fit changes with the scatter plot at http://argyll.epsb.ca/jreed/math9/strand4/scatterPlot.htm first. You may also go to www.wiredmath.ca for the link.

3. For the following data create an appropriate scatter plot, and determine the curve of best fit.
a. Women's 100 meter dash

| Year | Time (s) |
| :---: | :---: |
| 1928 | 12.2 |
| 1932 | 11.9 |
| 1936 | 11.5 |
| 1948 | 11.9 |
| 1952 | 11.65 |
| 1956 | 11.82 |
| 1960 | 11.18 |
| 1964 | 11.49 |
| 1968 | 11.08 |
| 1972 | 11.07 |
| 1976 | 11.08 |
| 1980 | 11.06 |
| 1984 | 10.97 |
| 1988 | 10.54 |
| 1992 | 10.82 |
| 1996 | 10.94 |
| 2000 | 10.75 |
| 2004 | 10.93 |

b. Wingspread of Birds

| Length <br> $(\mathrm{cm})$ | Wingspread <br> $(\mathrm{cm})$ |
| :---: | :---: |
| 123 | 350 |
| 120 | 300 |
| 175 | 250 |
| 78 | 222 |
| 124 | 157 |
| 75 | 128 |
| 17 | 27 |
| 18 | 50 |
| 48 | 71 |
| 62 | 75 |
| 38 | 76 |
| 48 | 86 |
| 47 | 100 |
| 52 | 100 |
| 51 | 120 |
| 56 | 120 |
| 55 | 127 |
| 70 | 150 |

c. Estimate of World's

Population

| Year | Population <br> (millions) |
| :---: | :---: |
| -400 | 160 |
| 0 | 170 |
| 200 | 190 |
| 400 | 190 |
| 600 | 200 |
| 800 | 220 |
| 1000 | 310 |
| 1200 | 360 |
| 1400 | 350 |
| 1600 | 545 |
| 1800 | 980 |
| 1900 | 1650 |
| 1920 | 1860 |
| 1940 | 2300 |
| 1960 | 3020 |
| 1980 | 4430 |
| 1990 | 5260 |
| 2000 | 6070 |

4. For each of the following scatter plots, create a table of values and draw a curve of best fit.
a.

b.
Areas and Depths of Seas

5. A few students got together, and decided to plot the number of hours they spent studying versus their performance on a test.

a. What does point A represent?
b. Draw a line of best fit on the scatter plot.
c. Based on this graph, is there a strong relationship between hours of study and a student's mark? Provide reasoning for your answer using a line of best fit.

## Direct Variation

When the ratio of two variables is constant, the variables are said to be in direct variation.

The points of this relationship form a straight line which passes through the origin.
Ex.
Salary Earnel

vs.

## Partial Variation

Similar to direct variation, the two variables involved change with a constant of variation. Only, in partial variation, there is a fixed starting value other than zero.

Ex.
Salary Earne


Expectations: i) identify the properties of linear relations ii) interpret the meanings of points on graphs and scatter plots. iii) construct tables of values, scatter
6. Alexander Helios and Cleopatra Selene are the twin children of Mark Antony and Cleopatra. For Alexander's first birthday, his parents gave him 4 gold coins, and continued giving him the same number of gold coins until his age of 15 . Meanwhile, Cleopatra has received 18 gold coins at birth, and 3 coins for every birthday after that, also until the age of 15.
a. Create a table of values representing the number of coins each twin had received since the day they were born. Then, plot both sets of points on the same graph.
b. Draw a line of best fit for each set of data, and write an appropriate equation for the line.
c. At which age will Alexander and Cleopatra have the same number of coins?
d. If they were to receive coins until the age of 25 , who would have more coins? How many more coins?
e. If Cleopatra were to receive 30 coins at birth, instead of the original 18 coins, who would have received more coins, in total, by the age of 21 ?


## Did You Know?

It is believed that the idea of Gravity and its extent were not developed (or even considered), until Sir Isaac Newton started working on the concept after being inspired by a falling apple in 1680's.

7. Create a scatter plot using the side lengths of equilateral triangles as your $x$-values and their perimeters as the $y$-values.
a. What relationship does the scatter plot show?

Write an equation for the relationship?
b. Use your graph to determine the perimeters of triangles whose side lengths are
i. 12 units long
ii. 45 units long
8. Two classes of 21 students were given 30 minutes each to pick apples. Below is a scatter plot showing how many apples each student picked, where the students are listed in the order they appear on the class list, starting with zero.
a. Draw a line of best fit for each class. Write the formula for the lines of best fit.
b. How many apples did students 4,6 , and 15 from class A, and student 9 from class B pick?
c. How many apples did students 17 through 20 from class B each pick?
d. A farmer requires 70 apples to make 21 pies. If only the first 5 students from each class give their apples to the farmer, which class would have picked enough apples?
e. How many more apples will the $20^{\text {th }}$ student from class A will have to pick to catch up to the $20^{\text {th }}$ student in class B?
f. In class A, 3 more students were added to the end of the class-list. If they continued picking apples at the same rate their classmates did, how many apples would each student pick?

9. Nhean and Alexis are partaking in a 7250 m bike race. Knowing Nhean is a slower biker, Alexis allows him a two-minute head start. Alexis's speed is $290 \mathrm{~m} / \mathrm{min}$, while Nhean's speed is $250 \mathrm{~m} / \mathrm{min}$.
a. On the same graph, plot each biker's distance travelled versus time.
b. Who won the race? By about how many minutes?


## CHALLENGE YOURSELF!

10. Dawn got into her car at 8 in the morning. She accelerated at a constant rate to $40 \mathrm{~km} / \mathrm{h}$ in 5 minutes. She drove for 15 minutes at $40 \mathrm{~km} / \mathrm{h}$ to the store, and then took 5 minutes to slow down to a stop. She parked in front of the store and was in there for 15 minutes. On the drive home from the store, Dawn accelerated to $70 \mathrm{~km} / \mathrm{h}$ in 5 minutes and drove toward home, but after 15 minutes, slowed down to a constant rate to 20 $\mathrm{km} / \mathrm{h}$ in 10 minutes due to traffic, and then drove at that speed for the next 40 minutes until she got home.
a. Graph the progress of Dawn's car using a time-speed graph.
b. What time did Dawn get home?

## EXTENSION

11. Newton tosses an apple straight up in the air from an initial height of 1 m , and recorded the height of the apple for 4 seconds.

Here is the table of his findings:

| Time (seconds) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height (m) | 1 | 9.75 | 16 | 19.75 | 21 | 19.75 | 16 | 9.75 | 1 |

a. Plot Newton's findings on a scatter, and draw an appropriate curve of best fit.
b. Will the apple ever reach a maximum height? Explain why, or why not. If so, what will be the maximum height, and when will it occur?


