Grade 9

Number Sense and Numeration: Roots and Pythagorean Theorem

Answers:

1. a. 10 b. 6 c. 1 d. 9 e. 12 f. 7
   g. 13 h. 15 i. 18 j. 30 k. 11 l. 20
   m. $\frac{4}{5}$ n. $\frac{19}{10}$ o. $\frac{3}{2}$ p. 0 q. $\frac{7}{5}$ r. $\frac{4}{5}$

2. a. 3 b. -5 c. 0.2 d. -9 e. $-\frac{3}{2}$ f. $\frac{5}{4}$
   g. $-\frac{1}{6}$ h. $\frac{4}{5}$ i. 0 j. $\frac{1}{4}$ l. $-\frac{1}{2}$ k. $\frac{9}{13}$

3. b. i. 3.2 ii. 2.2 iii. 2.8 iv. -3.1 v. 0.5 vi. -2.5

4. a. 5 cm b. 12 cm c. 7 cm d. 61 cm e. 15 cm f. 29 cm

5. a. 15 cm (3,4,5 multiple of 3)
   b. 24 cm (5,12,13 multiple of 2)
   c. 14 cm (7,24,25 multiple of 2)
   d. 20 cm (3,4,5 multiple of 4)
   e. 21 cm (7,24,25 multiple of 3)
   f. 29 cm (20,21,29 multiple of 2)

6. a. $x = 45 + 7 = 52$ (8,15,17 multiple of 3) and (7,24,25)
   b. $x = 54$ (3,4,5), $y = 120$ (3,4,5)

7. a. First determine that $25^2 = 625$, $20^2 = 400$ and $15^2 = 225$.
   Since $625 = 400 + 225$, the triangle is right angled at the vertex opposite the longest side of length 25.
   b. First determine that $37^2 = 1369$, $35^2 = 1225$ and $12^2 = 144$.
   Since $1369 = 1225 + 144$, the triangle is right angled at the vertex opposite the longest side of length 37.
   c. First determine that $26^2 = 676$, $24^2 = 576$ and $20^2 = 400$.
   Since $676 \neq 576 + 400$, the triangle is not right angled.
   d. First determine that $\left(\sqrt{2}\right)^2 = 2$ and $1^2 = 1$.
   Since $2 = 1 + 1$, the triangle is right angled at the vertex opposite the longest side of length $\sqrt{2}$. This triangle with sides of lengths 1, 1 and $\sqrt{2}$ is a right-angled isosceles triangle with angles of $45°$, $45°$ and $90°$.

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e. First determine that $2^2 = 4$, $1^2 = 1$ and $\left(\sqrt{3}\right)^2 = 3$.
Since $4 = 1 + 3$, the triangle is right angled at the vertex opposite the longest side of length 2. This triangle with sides of lengths 1, $\sqrt{3}$ and 2 is a right-angled triangle with angles of $60°$, $30°$ and $90°$.

f. First determine that $3.0^2 = 9$, $2.4^2 = 5.76$ and $1.8^2 = 3.24$.
Since $9 = 5.76 + 3.24$, the triangle is right angled at the vertex opposite the longest side of length 3.

g. First determine that $65^2 = 4225$, $63^2 = 3969$ and $16^2 = 256$.
Since $4225 = 3969 + 256$, the triangle is right angled at the vertex opposite the longest side of length 65.

h. First determine that $5^2 = 25$, $3^2 = 9$ and $\left(\sqrt{34}\right)^2 = 34$.
Since $34 = 25 + 9$, the triangle is right angled at the vertex opposite the longest side of length $\sqrt{34}$.

8. 65 m $\left[\text{diagonal} = \sqrt{33^2 + 56^2}\right]$

9. The longest stick would be the diagonal of the rectangular prism.
It's length is $d = \sqrt{12^2 + 5^2 + 84^2} = \sqrt{7225} = 85$ cm.
Therefore, the longest stick that can be placed inside the rectangular prism is 85 cm.
Alternatively, calculate the diagonal of the rectangular base of the prism.
It is 13 cm (5,12,13 Pythagorean triple).
Then calculate the diagonal of the prism, using the Pythagorean Theorem where $d = \sqrt{13^2 + 84^2} = 85$ cm.

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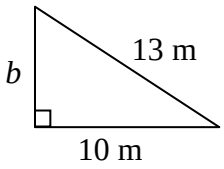
10. a. The top of the ladder is 12 m from the base of the wall (5,12,13 Pythagorean triple).

b.



Let b represent the height of the ladder above the base.
Using the Pythagorean Theorem

$$b^2 + 10^2 = 13^2$$
$$b^2 = 169 - 100$$
$$b^2 = 69$$
$$b = \sqrt{69}$$
$$b \doteq 8.3$$

Its descent from the previous position is $12 - 8.3 = 3.7$ m.
The ladder has descended another 3.7 m.

11. a. The volume of a cube is $V = x^3$, where x is the length of each edge.
Thus, $2744 = x^3$ and $x = \sqrt[3]{2744} = 14$.
Therefore, each edge has a length of 14 cm.

b. The diagonal has a length $d = \sqrt{14^2 + 14^2 + 14^2} = \sqrt{588} \doteq 24.2$ cm.

12. Using the formula $V = \frac{4}{3}\pi r^3$, $1435 = \frac{4}{3}\pi r^3$ so $r^3 = \frac{1076.25}{\pi}$.

Using $\pi \doteq 3.142$, we get $r \doteq \sqrt{342.537} \doteq 7$.
Therefore, the radius is 7 cm.

13. a. $\sqrt[3]{\frac{54}{16}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

b. $\frac{\sqrt[3]{27+64+125}}{\sqrt{1+8+27+64}} = \frac{\sqrt[3]{216}}{\sqrt{100}} = \frac{6}{10} = \frac{3}{5}$

c. $\frac{\sqrt{243}}{\sqrt{75}} = \frac{\sqrt{81 \times 3}}{\sqrt{25 \times 3}} = \frac{\sqrt{81} \times \sqrt{3}}{\sqrt{25} \times \sqrt{3}} = \frac{9}{5}$

14. 25π cm^2
The diameter is $AC = 10$ cm (3,4,5 multiple of 2).
Thus the radius is 5 cm.
The area of the circle is $A = \pi(5)^2 = 25\pi$ cm^2.

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15. 16

Since $S = \sqrt{x_1 + x_2 - x_3 - x_4}$ is a real number, then $x_1 + x_2 - x_3 - x_4 \geq 0$ which means $x_1 + x_2 \geq x_3 + x_4$.

Also, $x_1 + x_2 + x_3 + x_4 \geq 10$.

Therefore, $x_1 + x_2 \geq 5$.

If $x_1 + x_2 = 5$, then there are 4 ways to do so. $(x_1, x_2) = (1,4)$ or $(4,1)$ or $(2,3)$ or $(3,2)$

If $x_1 + x_2 = 6$, then there are 2 ways. $(x_1, x_2) = (2,4)$ or $(4,2)$

If $x_1 + x_2 = 7$, then there are 2 ways. $(x_1, x_2) = (3,4)$ or $(4,3)$

This gives a total of 8 ways.

Each case gives rise to 2 possibilities for x_3 and x_4 (either order).

Altogether, there are 16 possibilities.