Grade 9

Number Sense and Numeration: Algebra and Polynomials

Answers:

1.

Polynomial	Number of Terms	Coefficient of x	Constant
$5x$	1	5	0
$2x+4y$	2	2	0
$3x^2-6x+4$	3	-6	4
$2x+16$	2	2	16
$4z^3+3y^2-5x-10$	4	-5	-10

2. a. $2x, -3x$
   b. $-4g, -2g$
   c. $2x^2, -4x^2$
   d. $xy, 2yx$
   e. $5n^2m^2, (3nm)^2$
   f. $2.5n^3, 3.2n^3, \frac{1}{2}n^3$

3. a. $5x$
   b. y
   c. $14x^2-9$
   d. s^4+5s^2
   e. $-2n+5$
   f. $3x-5$
   g. $1.3x+0.2y$
   h. $4x^3+x^2$
   i. $2h+2$
   j. $5x-2y$
   k. x^2+x+1
   l. $-3x^2+4xy+9y^2$

4. a. $5(a+2)$
   $=5a+10$

   b. $-4(2-3x)$
   $=-8+12x$

   c. $2(x^2-4x+1)$
   $=2x^2-8x+2$

   d. $2(3x-1)-5(4x+2)$
   $=6x-2-20x-10$
   $=-14x-12$

   e. $4(5x-1)-(2x-3)$
   $=20x-4-2x+3$
   $=18x-1$

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f. $\frac{3}{2}\left(\frac{1}{3}a-\frac{2}{3}b\right)-\frac{3}{4}\left(\frac{1}{3}a+\frac{2}{3}b\right)$

$=\frac{1}{2}a-b-\frac{1}{4}a-\frac{1}{2}b$

$=\frac{1}{4}a-\frac{3}{2}b+8$

5. a. $56xy+28y$
   b. $10x^2-6x$
   c. $95x^3y+57x^2y+133xy$
   d. $-15xy^3-21x^2y^2$
   e. $-12x^3+8x^2y-4xy^2$
   f. $-3x^5y^7-2x^4y^7-2x^2y^6$

6. a. $A=(2x)(2x)=4x^2$
   b. $A=(x)(x)=x^2$
   c. Shaded area = area of large square – area of small square
   $=4x^2-x^2$
   $=3x^2$

7. a. Let x be the number of toonies. Then, $2x$ and $4x$ are the number of loonies and quarters respectively. So the total number of coins is $x+2x+4x=7x$.
   b. The value of the coins is $(2)x+(1)2x+(0.25)4x=5x$.
   c. If $x=2$, then $5\times 2=\$10$.

8. a. Initial parking fee $=\$12.50-\$1.50\times 6=\$3.50$.
   b. Let x be the number of hours, then Parking fee $=\$3.50+\$1.50x$.
   c. If $x=8$, then the parking fee is $\$3.50+\$1.50\times 8=\$3.50+\$12.00=\$15.50$.

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9. a. $5k+3=8$
$5k=5$
$k=1$

b. $4+7m=4m-2$
$7m-4m=-2-4$
$3m=-6$
$m=-2$

c. $15-3z=4z+8$
$-3z-4z=8-15$
$-7z=-7$
$z=1$

d. $2(x-3)-5=6$
$2x-6-5=6$
$2x-11=6$
$2x=17$
$x=8.5$

e. $-14b-5=12b+8$
$-14b-12b=8+5$
$-26b=13$
$b=-0.5$

f. $5(x+4)-10=5+4(x-1)$
$5x+20-10=5+4x-4$
$5x+10=4x+1$
$5x-4x=1-10$
$x=-9$

g. $14x-16=6+2x+2$
$14x-2x=8+16$
$12x=24$
$x=2$

h. $3(a-2)+5a=14-6(5-3a)$
$3a-6+5a=14-30+18a$
$8a-18a=-16+6$
$-10a=-10$
$a=1$

10. a. $3x+10=26-x$
$4x=16$
$x=4$

b. $EF=3x+10$ and $x=4$ so $EF=22$.
Since, $EF=GH$, then $EF+GH=22+22=44$ cm.

c. $AB+CD$ and $EF+GH$ are of same length.

d. $2y+5+6y+15=44$
$8y+20=44$
$8y=24$
$y=3$ cm.

11. Let the three consecutive numbers be $(x-1)$, x and $(x+1)$.

$$(x-1)+(x)+(x+1)=144$$
$$3x=144$$
$$x=48$$

$x-1=47$ and $x+1=49$

Therefore, the numbers are 47, 48 and 49.

12. Let x represent Charlie's age and $30-x$ be Jack's age.
So, in 5 years, Charlie's age will be $x+5$ and Jack's age will be $30-x+5$.
Thus, in 5 years, 3 times Charlie's age will be $3(x+5)$ which will equal Jack's age.

$$3(x+5)=30-x+5$$
$$3x+15=35-x$$
$$4x=20$$
$$x=5$$

and $30-x=25$.

Thus, Jack is 25 years old and 4 candles should be added to the cake.

13. a. Car Agency A: $C=\$30.00+\$1.25d$
Car Agency B: $C=\$20.00+\$1.50d$

b. Car Agency A: $C=\$30.00+\$1.25\times 30=\$30.00+\$37.50=\$67.50$
Car Agency B: $C=\$20.00+\$1.50\times 30=\$20.00+\$45.00=\$65.00$
To minimize cost, Mary should rent her car from Car Agency B.

c. Car Agency A: $C=\$30.00+\$1.25\times 150=\$30.00+\$187.50=\$217.50$
Car Agency B: $C=\$20.00+\$1.50\times 150=\$20.00+\$225.00=\$245.00$
For William's trip, Car Agency A would be cheaper to rent.

d. Let x be the distance travelled.

$$20.00 + 1.50x = 30.00 + 1.25x$$
$$1.50x - 1.25x = 30 - 20$$
$$0.25x = 10$$
$$x = \frac{10}{0.25}$$
$$x = 40$$

So, the cost to rent a car will be equal if 40 km is driven.

e. $C = \$30.00 + \$1.25 \times 40 = \$30.00 + \$50.00 = \$80.00$

14. Let the number of men be x and the number of women be y. The sum of the men's ages is $35x$ and the sum of the women's ages is $25y$.
Since the average age of the entire group is 31:

$$\frac{35x + 25y}{x + y} = 31$$
$$35x + 25y = 31x + 31y$$
$$35x - 31x = 31y - 25y$$
$$4x = 6y$$
$$\frac{x}{y} = \frac{3}{2}$$

So, the ratio of the number of men to the number of women is 3:2.

15. Let the sides of the rectangular solid be represented by a, b and c, then $ab = 32$, $bc = 24$ and $ca = 48$. Notice that each side is used in two surface area calculations. So, if we multiplied the surface areas together:
$ab \times bc \times ca = a^2b^2c^2 = 32 \times 24 \times 48 = 36864$.
The volume = $abc = \sqrt{a^2b^2c^2} = \sqrt{36864} = 192$ cm^3.

16. a. a^2 b. b^2 c. ab d. ab e. $(a+b)^2$

f. They equal to each other, which shows $(a+b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

17. a. $(3a+2b)^2 = (3a)^2 + 2(3a)(2b) + (2b)^2 = 9a^2 + 12ab + 4b^2$

b. $(2x-y) = (2x)^2 - 2(2x)(y) + (y)^2 = 4x^2 - 4xy + y^2$

c. $\left(\frac{1}{2}m + \frac{1}{3}n^2\right)^2 = \left(\frac{1}{2}m\right)^2 + 2\left(\frac{1}{2}m\right)\left(\frac{1}{3}n^2\right) + \left(\frac{1}{3}n^2\right)^2 = \frac{1}{4}m^2 + \frac{1}{3}mn^2 + \frac{1}{9}n^4$

d. Since there are 3 terms, and the formula only applies to binomials, we will substitute to make 3 terms. Let $d = a + b$. Now, the equation looks like a binomial.

$(d+c)^2$

$= (d)^2 + 2(d)(c) + (c)^2$

Substituting $(a + b)$ back for d:

$= (a+b)^2 + 2(a+b)(c) + c^2$

and applying the binomial expansion formula again we get:

$= (a)^2 + 2(a)(b) + (b)^2 + 2ac + 2bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Alternative Solution #1:

$(a+b+c)^2$

$= \left[(a+b)+c\right]^2$

$= (a+b)^2 + 2(a+b)c + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Alternative Solution #2:

$(a+b+c)^2$

$= (a+b+c)(a+b+c)$

$= a^2 + ab + ac + b^2 + ba + bc + c^2 + ca + cb$

$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

18. $(a+b+c+d)^2$

$= a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$