Grade 9

Analytic Geometry: Solving Systems of Linear Equations

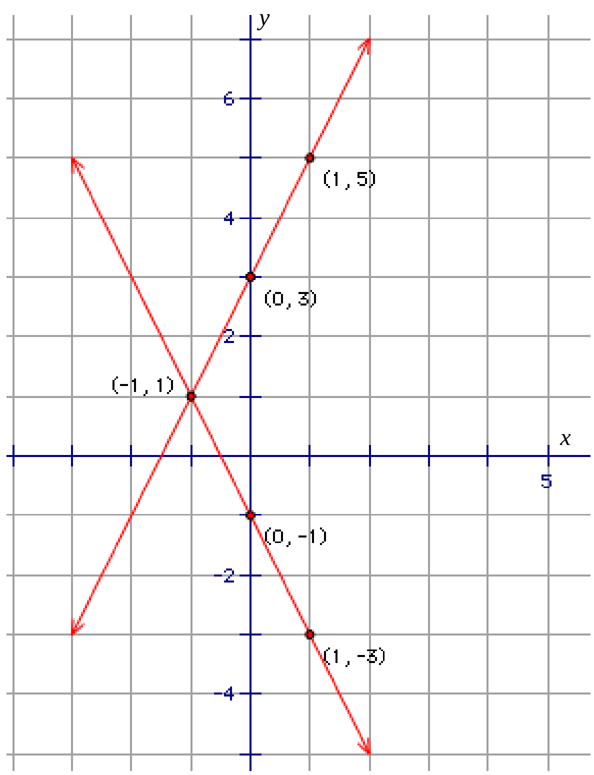
Play **System of Equations Basketball** first.
Click on http://www.math-play.com/System-of--Equations-Game.html to play.
You can go to *www.wiredmath.ca* for the link.

A **system of equations** is a collection of two or more equations with the same variables.

For example: $\left.\begin{aligned} y &= 2x+3 \\ y &= -2x-1 \end{aligned}\right\}$ is a system of equations in two variables, x and y.

When we **solve** a system of equations, we find values of x and y that satisfy both equations. If $(a,\ b)$ is the point of intersection, then when a is substituted for x in the equations, both equations will produce an answer of b for y.

A system of equations can be solved by graphing both equations on the same axes.



Example 1

Solve the following system of equations:

$y = 2x+3$

$y = -2x-1$.

Solution

Both equations are in slope y-intercept form. To graph each line, we plot a point corresponding to the y-intercept and then use rise and run to get a second point. Finally, join both points.

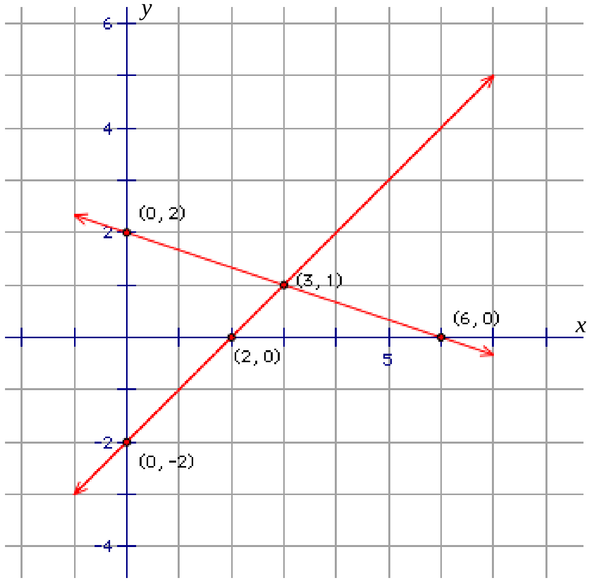
In this case, our first line has a slope of 2 and a y-intercept of 3. We get the first point (0, 3) and use rise of 2 and run of 1 to get a second point $(1,\ 5)$.

The second equation has a slope of – 2 and a y-intercept of -1. We get the first point $(0,\ -1)$ and use rise of -2 and run of 1 to get the other point $(1,\ 3)$.

Can you see where the lines cross? This is the point of intersection as well as the solution to this system of equations.

The solution to this system of equations is the point $(-1,\ 1)$.

Expectations i) determine graphically the point of intersection of two linear relations ii) interpret the intersection point in the context of an application.



Example 2
Solve the following system of equations:

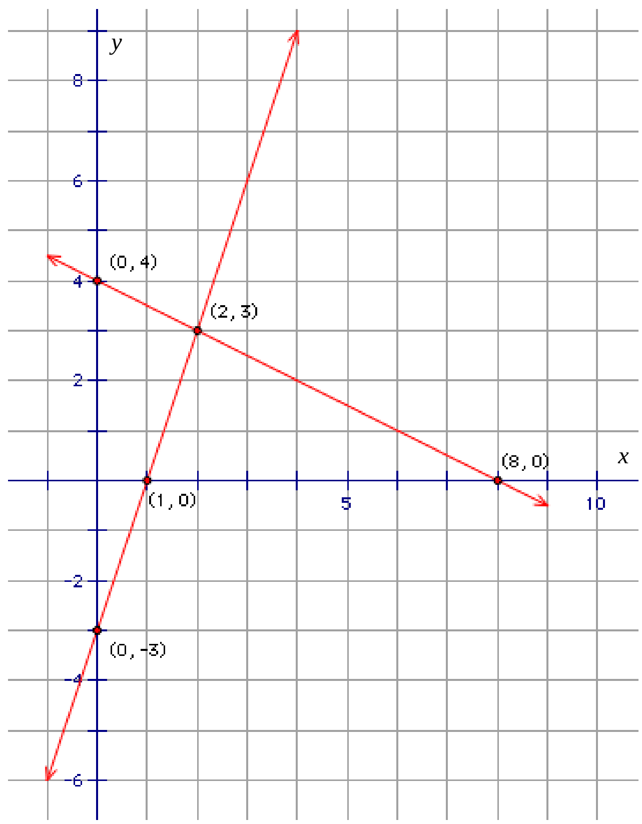
$$x + 3y = 6$$
$$x - y = 2$$

Solution
When equations are not given in slope *y*-intercept form, it is easier to use the *x* and *y* intercepts to obtain two points. To solve for the *x*-intercept, we let $y = 0$, and for the *y*-intercept, we let $x = 0$.
For $x + 3y = 6$, the *x*-intercept is 6 and the *y*-intercept is 2. These correspond to the points (6, 0) and (0, 2) on a graph.
For $x - y = 2$, the *x*-intercept is 2 and the *y*-intercept is -2. These correspond to (2, 0) and $(0, -2)$ on a graph.
Graphing these lines, we can see the solution is the point (3, 1).

Example 3
Solve the following system of equations:

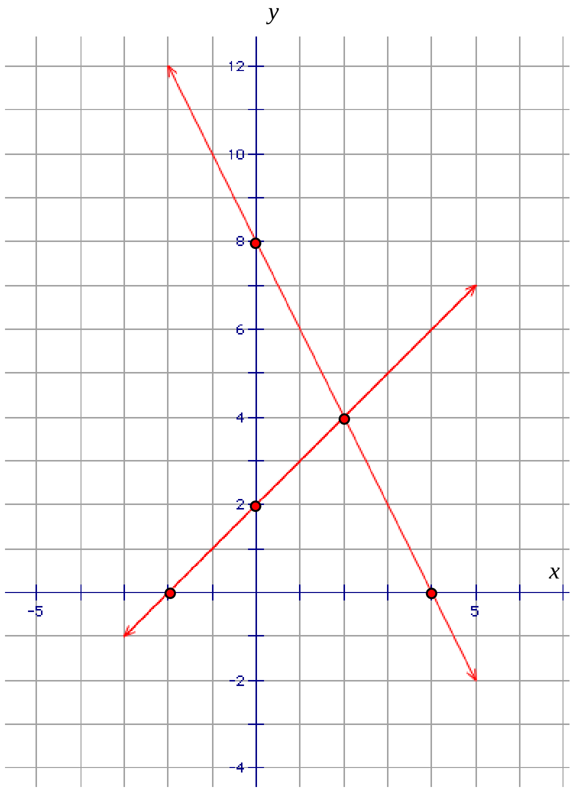
$$x + 2y = 8$$
$$y = 3x - 3$$

Solution
In the first equation, we obtain two points by using the intercept method.
For $x + 2y = 8$, the *x*-intercept is 8 and the *y*-intercept is 4. These correspond to (8, 0) and (0, 4) on a graph.
In the second equation, the slope is 3 and the *y*-intercept is -3. We get point $(0, -3)$ on the graph, and using rise of 3 and run of 1 to get another point $(1, 0)$.
Graphing these lines, we can see the solution is point (2, 3).

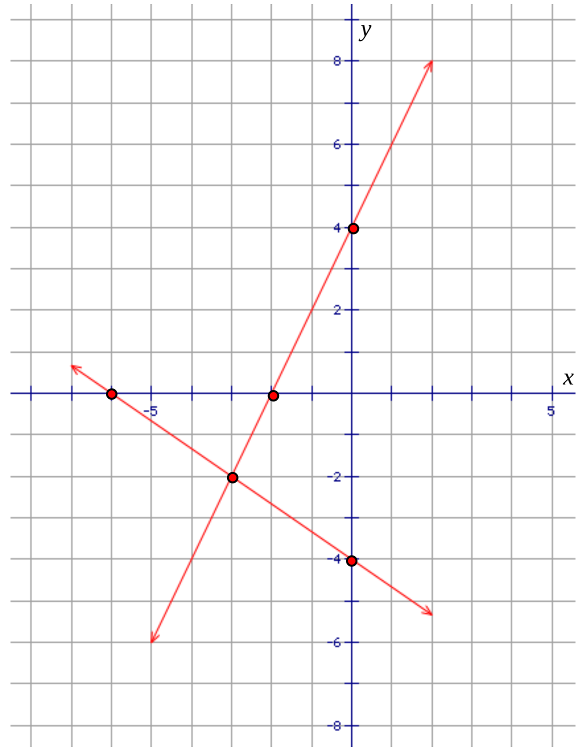


1. From the following graphs, determine the coordinates of the points of intersection.

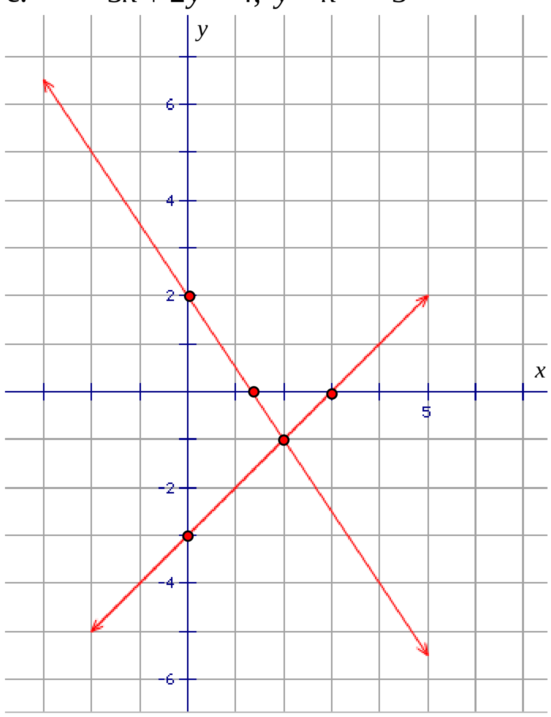
a. $y = x + 2,\ y = -2x + 8$



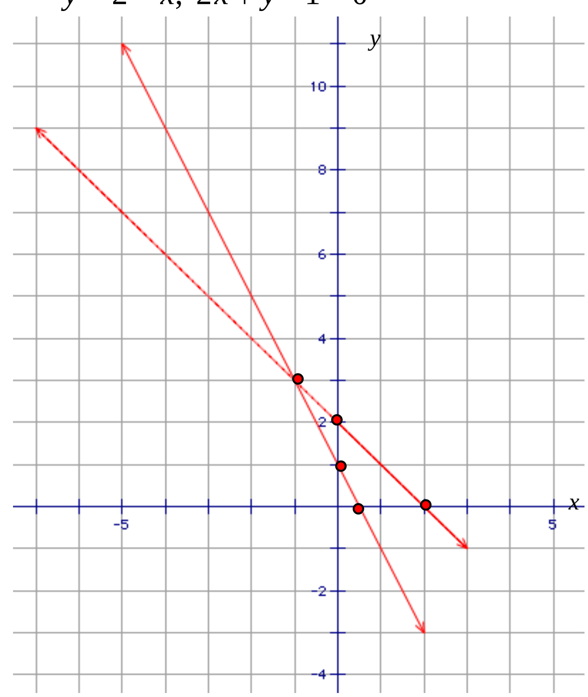
b. $2x + 3y = -12,\ y = 2x + 4$



c. $3x + 2y = 4,\ y - x = -3$



d. $y = 2 - x,\ 2x + y - 1 = 0$



2. a. Graph the following systems of equations. From the graph, determine the point of intersection.

i. $y = 6 - x$
$y = 2x + 6$

ii. $2x + 3y = 7$
$6y - x = 19$

iii. $y = 2x - 13$
$y = 2x + 3$

iv. $y = x + 2$
$y = 2x + 4$

v. $y = x + 2$
$5 = y + 2x$

vi. $y - 2x = -7$
$y - 3x = -12$

vii. $y - x = -2$
$y - 2x = -4$

viii. $6 - x = y$
$2y - x = 0$

ix. $y = x - 4$
$y - 2x = -9$

x. $y - 3x = 2$
$y = 3x + 5$

xi. $y = 8 - x$
$y = x - 4$

xii. $2x + 3y - 4 = 0$
$4x + 6y = 17$

b. From your results above, what characteristic do graphs share that have no point of intersection?

c. Plot your points of intersection on a grid. Label each so that it corresponds to the letter of its question. Connect the following points:
   i. a. to b.
   ii. a. to c.
   iii. e. to f.
   iv. g. to h.
   v. j. to k.

What letter do you see? If you did it correctly and you study hard, you'll be sure to get many of those in your near future.

Did you Know?
How far do you think the lead (actually graphite) in your pencils can take you? Well, it has been tested and a typical HB pencil can draw one continuous line that is 55 kilometres (34 miles) long! That is roughly the same as writing out 45 000 words! Can you draw a graph that shows a relationship between the length of a line you can draw and the number of pencils you must use?

3. Randy is trying to start his own business selling pencils. It costs Randy $5.00 to start his business. Randy buys each pencil from a manufacturer for 30¢ and sells each pencil for 50¢.
   a. Write an equation to represent Randy's costs (the amount he spends).
   b. Write an equation to represent his revenue (amount of money he receives).
   c. Graph the two equations from parts a. and b. on the same set of axes.
   d. From the graph, determine the point of intersection. What does this mean in this scenario?
   e. If Randy sells 50 pencils, what is his profit (Profit equals Revenue minus Cost or $P = R - C$)?
   f. If Randy lowers his price to 30¢, how many pencils must he sell to break even?

Expectations i) determine graphically the point of intersection of two linear relations ii) interpret the intersection point in the context of an application.

CHALLENGE YOURSELF!

4. The lines represented by the equations intersect at three points, which are vertices of a triangle. Calculate the area of the triangle formed.

$y = 7 - x$

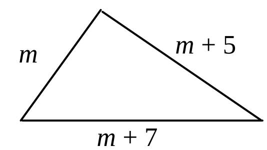
$y = 7 - 7x$

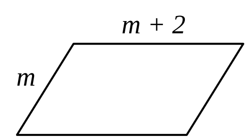
$y = 0$

A Slice of History

Algebraic solutions to solving systems of equations were formalized to the world by Carl Friedrich Gauss, but similar methods can be traced back as early as 150 AD in a Chinese book called *The Nine Chapters of Mathematical Art*.

5. On the right, two figures have the same perimeter.
   a. Write a system of equations to model the perimeters.
   b. Graph the equations of the perimeters on the same axes.
   c. Determine the coordinates of the point of intersection.
   d. Determine the perimeter of the triangle and the parallelogram when their perimeters are equal.





6. Jessica went to the supermarket to buy some fruit. When she got to the cashier, the clerk asked her how many of each fruit she had. She smiled and replied, “I bought four fewer oranges than two times the number of apples.” The cashier looked puzzled so Jessica added, “The number of oranges I bought is also the same as eight subtract two times the number of apples I bought.”
   a. Write a system of equations to model how much fruit she bought. Graph the line represented by each equation.
   b. From the graph, determine the point of intersection.
   c. How many apples and oranges did she buy?
   d. What is the sum of the two equations? How is this helpful?

7. For the following system of equations, find a value of k such that the lines are perpendicular to each other.

$kx + 2y - 1 = 0$

$2x - 3y + 5 = 0$

Don’t forget to try these math drills now! Go to www.wiredmath.ca for the link.

TRY THIS!

Systems of Equations Quizzes

http://www.quia.com/cz/10498.html?AP_rand=1900171386

Expectations: i) determine graphically the point of intersection of two linear relations ii) interpret the intersection point in the context of an application. *For more activities and resources from the University of Waterloo’s Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.*